Numerical Simulation of Jellyfish Swimming

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Abstract - Jellyfish are considered the most energetically efficient swimmers to have ever existed, so their propulsion mechanism can be researched to improve our own underwater vehicle designs. These unique animals need to be very efficient because of their nontraditional bodily components, and because most consume limited food while they prey passively during swimming. Jellyfish accomplish their efficiency through vortex propulsion. The contraction of a jellyfish's bell generates a vortex ring as it swims, which due to its axial symmetry we will simplify into two dimensions using two point vortices. We model these vortices using a system of differential equations, for which parameters can be selected to adjust their strength, location, and direction of rotation. To numerically solve the system, we use the fourth order Runge-Kutta method in MATLAB. The first goal of the project is to examine the jellyfish's propulsion and maneuvering mechanism, which involves creating simulations for various parameter values in the system of differential equations. Secondly, we study the material transport in the vicinity of jellyfish and its implication on food acquisition. Massless particles are inserted into the fluid flow to observe how the jellyfish and these particles in its environment move through the water as affected by the vortices. To work towards these goals, we have implemented bell shapes to represent moon jellyfish, Pacific sea nettles, cannonball jellyfish, and lion's mane jellyfish to provide a breadth of bell shapes and sizes. Each of these bell shapes serve as a barrier to material transport, so the jellyfish can capture particles from the surrounding environment. These results have significant implications for fields such as biomimetic engineering, enabling improvements to sub-aquatic vehicle efficiency and reliability, especially in cases when speed is not a priority.

Keywords : jellyfish propulsion; mathematical modeling

Mathematics Subject Classification (2020) **:** 92B05

1 Introduction

Jellyfish are one of the most energetically efficient swimmers ever studied. This efficiency is biologically crucial because of their limited and nontraditional anatomy, and many consume relatively limited food because they prey passively as they swim [7]. In order to achieve this efficiency, jellyfish create vortex rings in the water to propel themselves forward. Their swim cycle consists of a contraction phase which involves curling their bell margin inwards slightly to generate a vortex ring. The second part of their swim cycle is the relaxation phase in which the bell returns to its natural state and the jellyfish prepares to create the next vortex ring. Depending on the species of jellyfish there may also be brief pauses between these phases [10]. The vortex ring is pushed away from the bell margin, and the jellyfish is propelled in the opposite direction. If the vortex ring is uniform all the way around the circumference of the bell, then the jellyfish will move in a linear path, excluding the impact of any other current in the water. If one side of the vortex ring is stronger than the other, then the jellyfish will arc away from the stronger side of the vortex. After the jellyfish has created a vortex ring, its bell relaxes and expands again so that it is ready for another contraction to generate another vortex ring. These vortices have been previously examined by researchers using dye visualizations, which make the vortices visible in the water. This process also enables researchers to observe the vortex-vortex interactions over multiple swim cycles [8].

Not only do these vortices serve to propel the jellyfish through the water, but they can also interact with each other depending on the speed and shape of the vortices as determined by the jellyfish's bell shape. In general, jellyfish are categorized by their bell shape as either oblate or prolate based on the ratio of their bell height to width, also known as the bell's fineness ratio. Oblate jellyfish are those with a relatively low ratio of bell height to diameter, and they utilize rowing propulsion, whereas prolate jellyfish, those with a relatively high ratio of height to diameter, utilize jet propulsion [17]. Oblate jellyfish utilize vortex-vortex interactions to increase the distance traveled by the jellyfish through each swim cycle without any extra energy exerted by the jellyfish. The new vortex formed during each contraction is known as the stopping vortex. In every swim cycle following the first, the vortex from the previous cycle is still in the water and functions in the next cycle as the starting vortex. The stopping vortex pushes off the starting vortex, not unlike a swimmer pushing off a wall, allowing for passive energy recapture and significantly increases the distance that the jellyfish can travel in each swim cycle [10]. Prolate jellyfish are not able to take advantage of vortex-vortex interactions and are exclusively propelled by the starting vortex.

These vortices also have the ability to displace any particles in the water surrounding the jellyfish, some of which might be potential food for the jellyfish, such as plankton. In particular, *Aurelia aurita*, or moon jellyfish, consume microcrustaceans that may be dispersed throughout the water and get caught in the vortices created as the jellyfish swim [21]. Jellyfish will often position their tentacles in their vortex rings so that they can fully take advantage of this passive capture of prey. This method of collecting nutrients and plankton suspended in the water play such a large role in how a jellyfish sustains itself that there are observable differences in the structure of the oral arms of jellyfish depending on their bell shape, and therefore type of propulsion used [23].

When designing devices and vehicles that are meant to travel through water and other fluids, structures such as propellers are most often used, but they are significantly less efficient and maneuverable than evolved animals like jellyfish. Typically, when engineers utilize biomimicry to better emulate the efficiency of existing organisms, they attempt to reproduce the way fish swim, as propelled by fin-like structures. However, the motions of the jellyfish come with more benefits than those of fish, especially energetic efficiency. Although there are drawbacks of modeling propulsion methods after jellyfish motion, particularly that jellyfish do tend to be slower than animals with fin-like structures [7]. Despite this, by working to better model the swimming motion of jellyfish, we can improve the way we design propulsion mechanisms in underwater vehicles, particularly those that need to travel efficiently for an extended period of time and do not require high speeds.

In work by Marut et al. [17] and Yu et al. [28], researchers created biomimetic robots which attempted to increase the speed of the jellyfish robots instead of simply directly copying the swimming of the jellyfish. To do this, researchers modeled after a prolate jellyfish, *Sarsia tubulosa*, so stopping vortices were not used as part of the jet propulsion mechanism. By furthering research of jellyfish motion, particularly mathematical explanations of their propulsion, it will become easier for engineers to develop jellyfish-inspired robots modeled after different species to prioritize specific benefits of vortex propulsion.

The main goal of the project is to model the swimming motion of jellyfish using a system of differential equations. The mathematical model introduced and analyzed in this work details how point vortices yield different swimming behaviors for a rigid jellyfish. This model will build off a system of differential equations from previous research and will be solved numerically suing the Fourth Order Runge-Kutta (RK4) method. We utilized a numerical method because the system of differential equations does not have an analytical solution. RK4 uses four sub-calculations over each time step, which we can then combine to generate a more precise approximation than other numerical methods and we can accurately determine the approximate location of the jellyfish over a chosen interval of time. Our approach will also include massless particles as they are moved by the motion of the jellyfish. These particles will be approximated as massless an have no self-propulsion.

One of the main components that is incorporated into the model through this project is a bell which functions as a barrier to material transport. We experiment with different shapes of bells to investigate the impact on collection of particles into the jellyfish's bell. The bell shapes are representative of a variety of sizes of jellyfish as well as a mix of prolate and oblate jellyfish. Specifically, we consider moon jellyfish (*Aurelia aurita*), Pacific sea nettles (*Chrysaora fuscescens*), cannonball jellyfish (*Stomolophus meleagris*), and lion's mane jellyfish (*Cyanea capillata*), all of which are members of the class *Scyphozoa*. For each of these species, we can adjust the parameter values used in the differential equation model to generate videos simulating each species' motion and ability to capture particulate matter from their surroundings. When the species is selected, the video output will be designed to match the size and shape of the jellyfish species' actual characteristics. The parameters in these simulations are specifically chosen to mimic videos of live jellyfish swimming, including changes in direction of motion. We can further investigate the propulsion of these actual jellyfish in the videos through additional visual components in the simulated videos, such as a vector field.

2 Background

2.1 Jellyfish Morphology

The main part of the jellyfish, from which it propels itself, is called the bell. The edge around the periphery of the bell is called the bell margin, which curls under as the jellyfish swims to generate a vortex ring in each swim cycle [9, 24]. In this contractive motion, the vortex ring is pushed away from the bottom of the bell, thus pushing the jellyfish in the opposite direction [7]. The bell and the vortex ring it creates are axially symmetric, so in the model we simplify to a two-dimensional cross-section. Our two-dimensional cross-section produces point vortices, which are characterized by rotational motion spiraling outward from the vortex's center. An example of this motion can be seen in Figure 1. We then examine the propulsive method of the jellyfish as counter-rotating two point vortices, one of either side of the bell margin. The shape of the bell will be determined by a function specialized to the anatomy of the species selected. The vortex strength is also specified based on the chosen bell and its size.

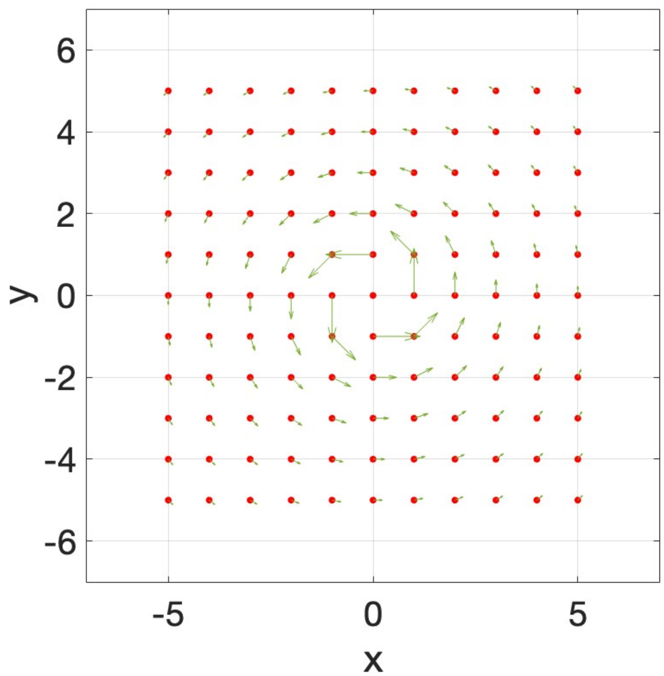

Figure 1: The figure shows a vector field representation of a point vortex with counter-clockwise rotation centered around the point $(0, 0)$ with $\Gamma = 1$.

Two other prominent pieces of the jellyfish's anatomy are oral arms and tentacles, but the presence of these depends on the species and their most common food sources. Both of these features are predominantly to help the jellyfish capture food, especially larger pray such as fish [11]. Some jellyfish, such as lion's mane jellies or sea nettles, have tentacles that can be many feet long to aid in capture of fish. Other species, such as moon jellies, do not have tentacles, but instead just oral arms, and consume primarily smaller prey such as plankton [24]. In this project, we model all jellyfish to have no tentacles. Although tentacles and oral arms do have a pronounced effect on the locomotion of jellyfish [14, 18], we are modeling jellyfish swimming without tentacles for simplicity. The simulations prioritize showing the the bell, as it is the source of the swimming mechanism of the jellyfish and also clearly indicates the location of the jellyfish as well as what direction it is traveling.

2.2 Included and Excluded Jellyfish Species

The animals that we classify as jellyfish come from four taxonimic classes, *Scyphozoa*, *Cubozoa*, *Staurozoa*, and *Hydrozoa*. For modeling purposes, we will focus on the species in the class *Scyphozoa*, which are often called "true jellyfish" due to their anatomy. Members of the class *Cubozoa*, commonly called box jellyfish, do not rely as heavily on the vortices generated by their bell contractions for capture of their prey. For example, Caribbean box jellyfish (*Tripedalia cystophora*) stun their prey with venom in order to capture larger self-propelled prey such as crabs and fish [3]. Additionally, many species in the classes *Staurozoa* and *Hydrozoa* do not have traditional jellyfish anatomy, particularly bells, and in such cases do not use the same method of propulsion as do *Scyphozoans*. The class *Hydrozoa* includes the species Portuguese Man O' War (*Physalia physalis*), which is frequently not classified as a jellyfish because of its lack of ability to propel itself through the water. Instead, it can only move as a result of oceans currents and winds [26] and does not have a bell capable of performing the kinds of contractions studied here. Other animals colloquially referred to as jellies include sea walnuts (*Mnemiopsis leidyi*), which belong to the class *Tentaculata*. Notably, while jellyfish belong to the phylum *Cnidaria*, sea walnuts are of the phylum *Ctenophora*. Again, this species is not capable of propulsive contractions to perform the swim cycle described and modeled here, but instead it moves as a result of combs on the lobes of their bell, and consequently their mechanism of motion is independent of their feeding mechanism [6].

As a result of these differences in propulsion and feeding among classes and species of jellyfish, we will focus exclusively on *Scyphozoans* throughout this project. Limiting our breadth of study allows us to include only species that use vortex propulsion in such a way that it impacts their foraging and consumption behaviors. Two of the most commonly recognized jellyfish in the class *Scyphozoa* are moon jellyfish (*Aurelia aurita*) and Pacific sea nettles (*Chrysaora fuscescens*). Moon jellyfish in particular are often the subject of propulsion modeling research because their anatomy is easy to observe with minimal tentacles [10]. Species such as moon jellyfish (*A. aurita*), Atlantic sea nettles (*Chrysaora quinquecirrha*), Bay Nettles (*Chrysaora chesapeakei*), lions mane jellies (*Cyanea capillata*), cannonball jellies (*Stomolophus meleagris*), spotted lagoon jellies (*Mastigias papua*), and blue blubber jellies (*Catostylus mosaicus*) are examples that all have the traditional anatomy that most people consider to be a classic jellyfish shape [15, 25, 19, 5, 1, 2, 13]. These species all have a bell which has a profile similar to a flat, wide hemi ellipse when expanded, and when their bells contract, the cross section is more similar to a major segment of a sphere. The flatness of the ellipse formed in expansion is largely dependent on the species of jellyfish. For example, lion's mane jellies' bells flatten almost entirely to a circle at full expansion, whereas other species such as the cannonball jelly still appear more so hemispherical throughout all phases of their swim cycle [15, 5].

However, not all *Scyphozoans* necessarily have a bell that perfectly satisfies the above description. For example, two species that belong to this class but do not quite have this characteristic bell shape are the bumpy jelly (*Stellamedusa ventana*) and the upside down jelly (*Cassiopea xamachana*). The former has a bell that is wider and flatter at the top, which creates a cross section which more so resembles a rounded square than a circular or elliptical shape [4]. Upside down jellyfish's bells appear to be inverted from the typical elliptical shape when fully expanded, and they typically pulse with their bell resting downwards onto the sea floor. Their bell margin arcs under creating a more traditional shape in contraction, but the top of their bell is still slightly concave inwards [27].

Generally, *Scyphozoans* consume zooplankton as at least one component of their diets, and for some it is the majority of what they prey on [13]. Examples of species that consume predominantly or exclusively consume plankton include the upside down jelly (*C. xamachana*), cannonball jelly (*S. meleagris*), and blue blubber jelly (*C. mosaicus*) [5, 2, 27]. For the most part, jellyfish such as these rely on zooplankton that do have some minimal capacity to propel themselves through the water, such as red durum larvae or crustacean zooplankton [5, 13]. Some jellyfish also use their vortices in conjunction with their tentacles to capture larger prey. Tentacles can be placed in the path of the vortices for effective capture of prey, where they can sting and then consume prey. For example, Atlantic sea nettles (*C. quinquecirrha*) mostly consume sea walnuts (*M. leidyi*) [13]. Due to their size, lion's mane jellyfish (*C. capillata*) are able to eat a large variety of prey such as small fish, crustaceans, immature moon jellyfish (*A. aurita*), and copepods [15]. In this project, we are most concerned with jellyfish that consume plankton as some part of their diet, as these microorganisms can typically be approximated as massless and do not affect the motion of the jellyfish. These microorganisms have either no capability for self-propulsion, or it is approximated as negligible compared to the vortex dynamics of the jellyfish.

2.3 Modeled Species

As described above, we focus exclusively on jellyfish of the class *Scyphozoa*. Making this distinction enables us to utilize certain commonalities in the jellies' anatomy when developing the code which generates the simulations. The species of moon jellyfish and Pacific sea nettles are chosen largely due to a availability of photographic and videographic materials, which aid in parameter adjustment and bell definition. The moon jellyfish in particular has significant prior research done on its propulsion method, which will aid in the development of a well-informed model. The specifics of developing the speed of motion in the model is based on observations of jellyfish through videos, and the selected bell shapes are largely adapted from actual photos of jellyfish, when paired with the limited available measurement data. The other two selected species are the cannonball jellyfish and lion's mane jellyfish due to the distinctly different bell shapes and sizes they possess, which provide an important contrast to the two previously discussed species. However, they both still consume plankton and possess the necessary anatomical components to make sense in the model.

We will use existing videos of jellyfish for comparison, both in the development of the model as well as in presentation of the complete video output of the code. The simulated bell shapes are created through visual analysis of photographs of different jellyfish to determine what mathematical functions approximately define the barrier of the bell. Particles can be incorporated into the model by approximating them as massless because their effect on the vortices is negligible compared to how significantly the vortices affect these particles. These particles will predominantly simulate plankton, under the assumption that they are not propelling themselves through the water.

3 Methods

3.1 Point Vortex Equations

The vortex ring created by the jellyfish can be modeled using a system of differential equations summed over each vortex in the model. Due to the axial symmetry of the jellyfish and vortex ring, we will simplify the problem to a two-dimensional cross section of the bell, which can be represented as two point vortices, one on either side of the bell margin in the cross section. Fluid dynamics have been modeled extensively in the field of physics, so the set of differential equations used for modeling point vortices are well-known in existing literature of mathematics and physics [22]. A system of two differential equations is needed in order to define each two-dimensional point vortex, as shown in the system of equations below [20]. However, since these equations do not have an exact analytical solution, it will be necessary to use the RK4 numerical method to solve for the coordinates of the location of the jellyfish at different time steps over the interval.

The system of differential equations used in this project, adapted from the Helmholtz-Kirchoff equations [20], calculates motion due to point vortices assuming no viscosity of the surrounding fluid:

$$\frac{dx_i}{dt} = \sum_{k=1}^{N} \left(\frac{1}{2\pi}\right) \frac{-(y_i - \tilde{y_k})\Gamma_k}{(x_i - \tilde{x_k})^2 + (y_i - \tilde{y_k})^2 + \delta} \tag{1}$$

$$\frac{dy_i}{dt} = \sum_{k=1}^{N} \left(\frac{1}{2\pi}\right) \frac{-(x_i - \tilde{x_k})\Gamma_k}{(x_i - \tilde{x_k})^2 + (y_i - \tilde{y_k})^2 + \delta} \tag{2}$$

The parameters included in the system allow for variation in the motion of the jellyfish. By varying the parameter values, we can control the current location of each vortex after every iteration of the numerical method, as well as the strength of rotation of each vortex. The strength and direction of the motion of the vortices, called their vorticities, can be set to change at a particular time to alter the direction of motion. Also included in the equations is a smoothing parameter, δ, in the denominator to prevent undefined vectors from dividing by zero. We also include particles in the model to represent potential prey in the jellyfish's environment, like plankton or microcrustaceans [21]. These points also provide a useful visual for how the water around the jellyfish is flowing. The code is designed such that adding points or altering their starting locations is straightforward, which allows the user to highlight the ability of the jellyfish to prey passively as it swims. As a result, the system includes two differential equations for each vortex and particle because the x and y coordinates are computed separately using equations 1 and 2 respectively. Each equation is calculated using a summation to ensure that both vortices impact each point and each other. This results in a system of differential equations which is potentially very large and has no analytical solution, meaning a numerical method is required for solving. The numerical solving method selected for this project is the Fourth Order Runge-Kutta (RK4) method, which is able to produce very small errors without requiring a greater number of time steps. The RK4 calculations are computed for each differential equation at every time step. For the majority of simulations, the size of each time step is 0.1 seconds, which produces smooth animations without being overly computationally expensive. When vectors are incorporated into the simulation, the time step is increased 0.13 seconds for the sake of visual clarity and continuity.

Each equation is summed over N points within the system, which includes all massless particles as well as the vortices themselves. The coordinate pairs of the particles and centers of the vortices are represented by x_i and y_i. Each pair of $\tilde{x_k}$ and $\tilde{y_k}$ represents the location of the center of a vortex. Each (x_i, y_i) pair is a massless point or vortex being propelled, while each $(\tilde{x_k}, \tilde{y_k})$ pair represents a vortex, allowing particles to be propelled by the vortices as well as enabling the vortices to propel each other. The parameter Γ_k controls the strength and direction of each vortex's rotation, or they can be set to switch values at a particular time in the simulation to change the jellyfish's path. For example, in Figure 1, the point vortex has a positive Γ_k, indicating counterclockwise rotation, with a magnitude of 1. The smoothing parameter δ is set to a constant value of 0.1 across all simulations to prevent the denominator of any of the differential equations from being equal to zero.

3.2 Coding Methodology

The parameters included in the planned system of differential equations allow for varying of the location of the vortices throughout the simulation, as well as varying their vorticities. While there are other physical factors which do play a role in the motion of the jellyfish, many of them will not be significantly examined in this project.

The approximate solution for the differential equations is based on the numerical method known as Fourth Order Runge-Kutta (RK4). Although each time step of RK4 does take longer to compute than some other methods in numerical analysis, it allows for much greater accuracy with significantly fewer time steps. We implement the RK4 method directly by developing code in MATLAB, which is an integrated development environment by MathWorks® designed specifically for writing code in mathematics which utilizes matrices [16]. The code for this project was developed in version R2023a. MATLAB is particularly convenient in this application because it provides a large library of mathematical functions and simply handles the arrays of points required for the different parts of the model. Although MATLAB has a built-in ode45 for numerical solving of systems of differential equations, instead we implement a function to performing RK4 computations explicitly in a MATLAB script, which allows for greater flexibility and customization within the code.

Within MATLAB, individual frames of a video can be created and then compiled into a single mp4 file. As a result, the process to create several videos with different parameter values for the differential equations is straightforward. MATLAB also provides the ability to add vector visualizations into each frame of the video, which can help give the viewer better understanding the fluid dynamics motivating the movement of the jellyfish.

We use existing videos of jellyfish for comparison, both in the development of the model as well as in presentation of the complete video output of the code. Photographs of jellyfish are also used to examine different bell shapes and determine what mathematical functions are best to define the barrier of the bell. Specifically, the model can accommodate both prolate and oblate bell shapes to demonstrate the differences in how the two propel themselves through the water.

Particles are included in the area surrounding the jellyfish to show how food like plankton is moved as a byproduct of the jellyfish's propulsion. The points are treated as massless because their effect on the surrounding fluid, and therefore, the jellyfish, is negligible compared to how significantly the jellyfish affects these particles. We add vectors in the background of the visualization to better demonstrate the flow propelling the jellyfish and any particles in the surrounding environment. These simulations are developed for four slightly different bell shapes, each inspired by the bell of some existing species of jellyfish. The different bell shapes can then be compared for their ability to capture particles from the water as they are affected by the vortices of the jellyfish.

3.3 Description of MATLAB Code

In order to generate a jellyfish bell with consistent properties throughout the various simulations, we define a class to create a bell object with specific functions defined within it. When a bell object is initialized, the user must specify which species is being modeled for a particular simulation, and the class will assign the shape of the bell accordingly. The properties of the bell include four row vectors separately containing the x and y coordinates of all the points composing the bell, and those that are considered to have been captured by the jellyfish. The class also contains the function which is responsible for translating and rotating the bell to its new location as affected by the point vortices. After each time step a particular function in the class is called which checks if any points are close enough to the bell to be considered captured, in which case it adds the current coordinates of the point to the appropriate row vectors. The massless points that are captured are translated and rotated along with the bell in each time step.

The main script is responsible for all differential equation computations with RK4, defining and tracking parameters representing the vorticity or the locations of the centers of the vortices, and for the plotting of figures and video generation. In the definition of these figures, the code selects the size of the axes on which to plot the jellyfish based on its species, as the different species have potentially quite different sizes. Additionally, the main script also defines and translates massless points over the course of the simulation. At the beginning of the simulation, 700 massless points are initialized into a rectangle which spans from $[-30, 30]$ in the x direction and $[-30, b]$ in the y direction, where b equals the minimum of the initial y-coordinates the bell minus 0.5. This array of points is sufficiently large to fill the space below the jellyfish for any of the included species. The massless points in the simulation are moved by the vortices, but do not affect the motion of the jellyfish or each other in the simulation.

Another modified script is used for generating videos that have vectors in the background. The vectors are established on a mesh grid, which spaces the initial points of the vectors by 1.5 units in the x direction and 1.57 units in the y direction. When dealing with vectors in the background, the time step is increased slightly to 0.13 seconds.

The location of the bell is determined by the location of the point vortices at the end of each time step. In order to preserve the shape of the bell, the vertical distance moved due to the vortices is calculated as well as the angle between the line segment connecting the two vortices and a horizontal line. Based on the calculated values, the coordinates representing the bell are then translated down and rotated using a rotation matrix.

4 Results

Bell shapes are modeled for four species of jellyfish:

- Moon jellyfish (*A. aurita*): $y = -\sqrt{4 - \frac{4x^2}{3.5^2}}, x \in (-3.5, 3.5)$,
- Pacific sea nettle (*C. fuscescens*): $y = -\sqrt{30 - 0.8267x^2}, x \in (-6, 6)$,
- Cannonball jellyfish (*S. meleagris*): $y = 1.5 \pm \sqrt{3.5^2 - x^2}, x \in [-3.17, 3.17]$, and
- Lion's mane jellyfish (*C. capillata*): $y = \frac{x^4}{2000} - 5, x \in [-10, 10]$.

These shapes can be seen in Figure 2 and were developed through the examination of photographs and videos of jellyfish of these respective species. As depicted in the figure, these bells are all of varying shapes, some oblate and some prolate, and notably they are also of dramatically different size. As a result, the type of jellyfish selected also automatically dictates the size of the axes on which the jellyfish, and any other additional features, will be plotted over the course of the simulation. The axes are scaled in inches to represent an average-sized jellyfish for each species. The gamma values for the simulations are selected to be of greater magnitude for larger jellyfish in order to produce similar movements across the species. Each time the simulation is run, the bell is defined with one of these shapes, and the shape is then translated and rotated according to the differential equations.

The most basic simulations simply show the jellyfish's bell being translated over time due to the motion of the vortices. In these videos, two main behaviors can be simulated, these being either motion is a straight line or in an arc, or later a combination thereof. Motion in a straight line can be observed in Figure 3, which shows a moon jellyfish bell being translated directly downwards. Motion such as this results whenever the values for the vorticity parameter of the two point vortices are of equal magnitudes, whereas when

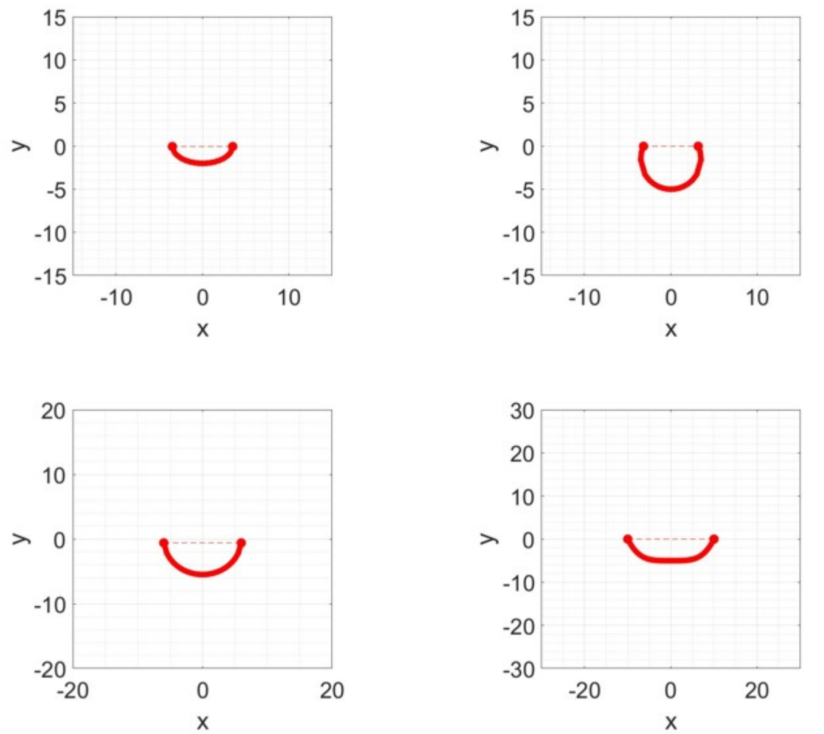

Figure 2: The four options for bell shapes that can be selected in the model. Clockwise from the top left image, moon jellyfish, cannonball jellyfish, lion's mane jellyfish, and Pacific sea nettle.

one vortex is stronger than the other, the resulting jellyfish behavior is as shown in Figure 4. Here, we observe the same bell shape moving in an arc to the left because the vortex associated with the left side of the bell is stronger than the right vortex.

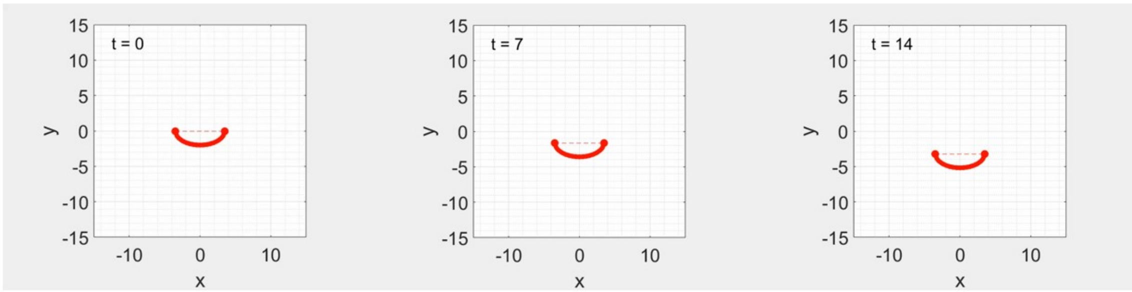

Figure 3: A simulation of a moon jellyfish swimming straight downward. $\Gamma = [10, -10]$

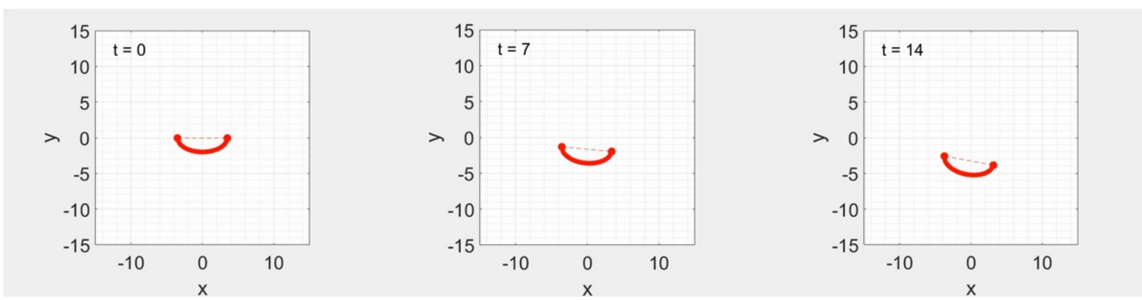

Figure 4: A simulation of a moon jellyfish swimming in an arc to the left. $\Gamma = [12, -8]$

Additional simulations model the vorticity, Γ, as defined by a piecewise function based on time. For example, in Figure 5, we can see frames from a simulation with a Pacific Sea nettle, designed to match the direction of motion of a video of a live jellyfish. The first ten seconds of the simulation the vorticity for the right hand vortex has a greater magnitude, causing the Pacific sea nettle to arc to the right. However, after ten seconds, the values for the vorticity parameters for each vortex are changed, resulting in the left vortex being stronger than the right, so that by the end of the simulation the jellyfish has come back to a level position, similarly to how it was initialized. These values for the gamma parameter were chosen to mimic a video of an actual Pacific sea nettle as it swims through the water because as most jellyfish swim they will not hold constant strengths around the circumference of their vortex ring over several contraction cycles.

Simulations are also created including vectors describing motion in the background of the video. These vectors are presented to better illustrate how the vortices are motivating the motion of the jellyfish. They are all spaced approximately 1.5 units away from each

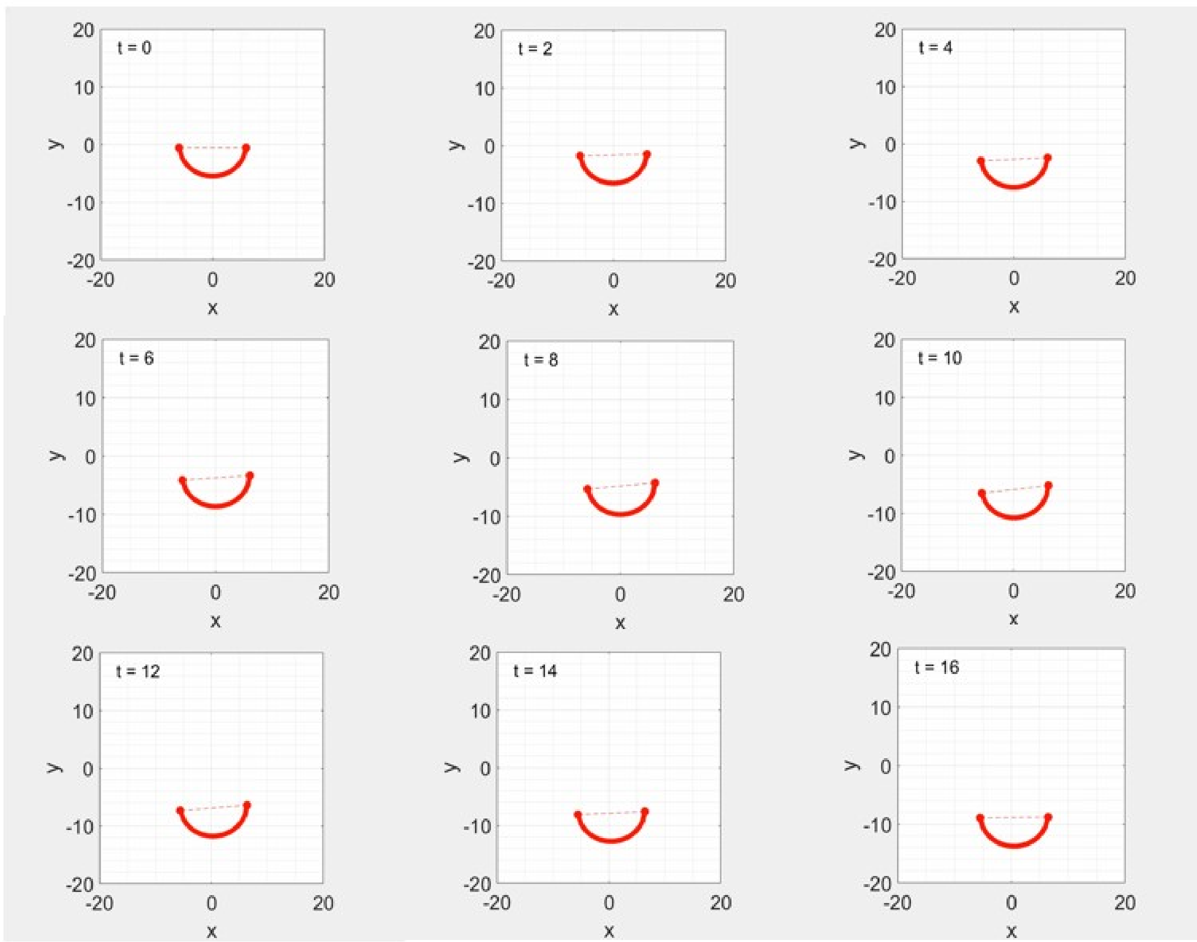

Figure 5: A simulation of a Pacific Sea nettle swimming in an arc to the right, and then arcing back to the left. $0 \leq t \leq 10 : \Gamma = [35, -45], t > 10 : \Gamma = [45, -30]$

other in each direction. The magnitude and direction of each vector is updated for each frame of the video. When the jellyfish is traveling in a straight line, the vectors are symmetric across the y-axis. When the jellyfish is traveling in an arcing motion, then the vectors are larger around the vortex opposite the direction in which the jellyfish is arcing. Such an arcing motion can be observed in Figure 6, in which a cannonball jellyfish arcs to the right, so the vectors around the right vortex are of greater magnitude. Throughout the frames shown in this figure, the vectors can be observed to cyclically increase and decrease in value to represent the distinct, repeating contractions of the jellyfish's bell over time.

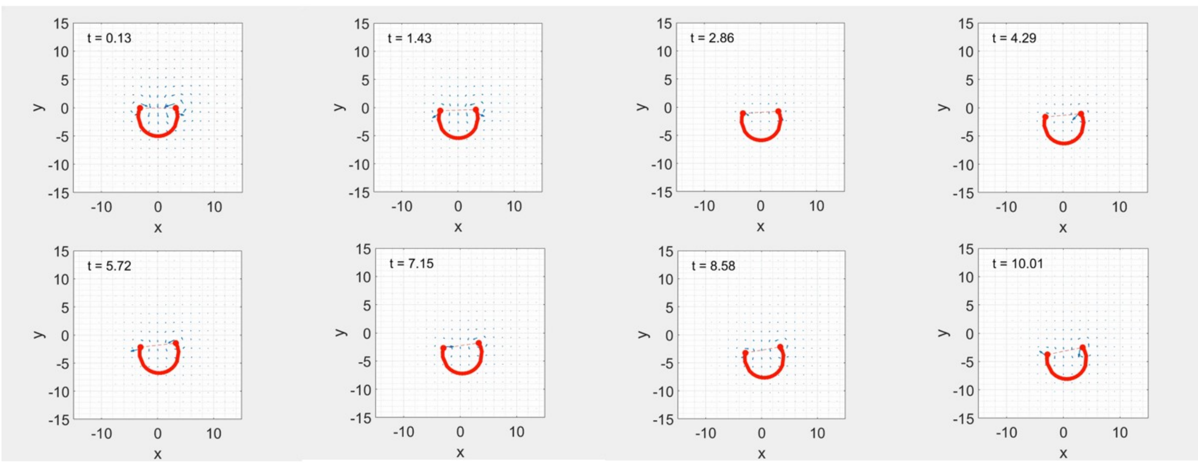

Figure 6: A simulation of a cannonball jellyfish swimming in an arc to the right with the resulting velocity field in the background. $\Gamma = [10, -15]$

Simulations can also be created incorporating points into the motion by approximating them as massless, meaning they do not affect each other's movement or the propulsion of the jellyfish. Seven hundred points are initialized at the beginning of the code randomly using a uniform distribution across a rectangle half a unit below the bell of the jellyfish. The same number of points are initialized regardless of the species of jellyfish selected, but some points may be outside of the plotting window for the video. During the simulation, the massless points are moved by the differential equations for the vortices of the jellyfish. Usually, the vortices pull the massless points up, around, and in to the bell of the jellyfish over time, as is shown with a lion's mane jellyfish in Figure 7. If the points collide with the bell, then they change from blue to cyan and are considered captured, and the bell class moves them along with the structure of the bell. At this point, if these points are microorganisms such as plankton, then the jellyfish would be able to consume them.

A similar simulation is shown in Figure 8, which depicts a moon jellyfish capturing massless points which swimming in an arc to the left. Notably, the difference in the strength of the vortices can affect which side of the bell captures the massless points. In the twenty five-second time interval for the simulation, all the points which are marked as captured touched the bell due to the left-hand vortex, which has a significantly greater gamma value. As a result, all the points that are captured came from the area to the left of the bell, and once inside end up on the left side of the jellyfish's bell.

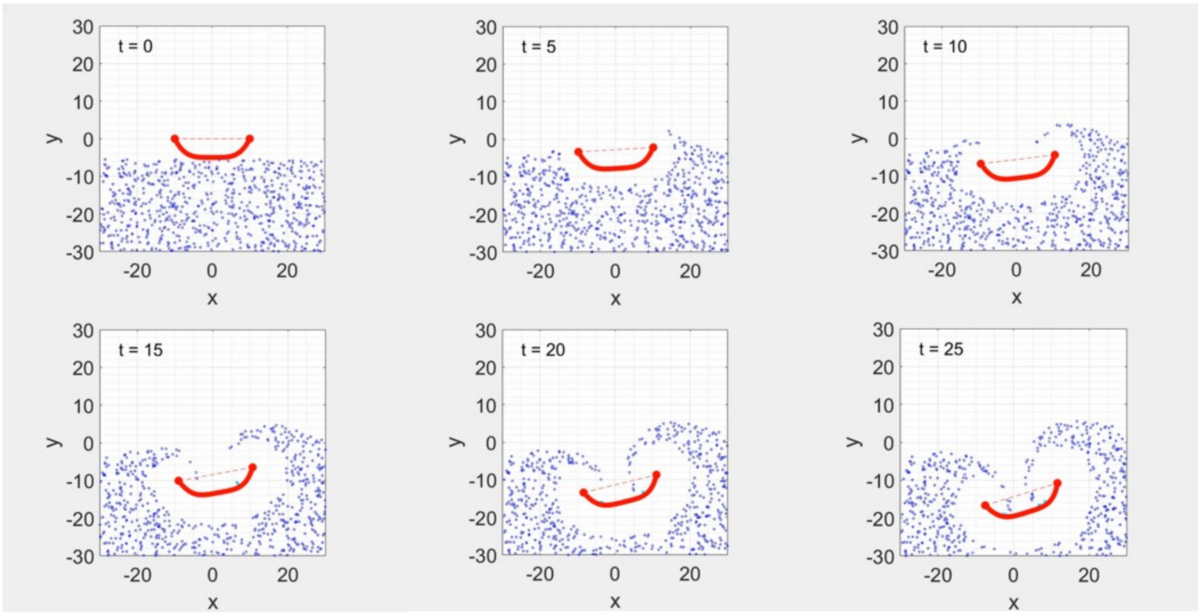

Figure 7: A simulation of a lion's mane jellyfish swimming in an arc to the right with randomly distributed massless points in the vicinity. $\Gamma = [55, -85]$

Both of these features, vectors as well as massless points, can be incorporated into the same simulation, however this leads to reduced visibility of either feature, even when the color are altered. The massless points and the vectors can be used for visualizations with any of the four studied jellyfish species.

5 Discussion and Conclusions

In this project we used a system of differential equations and the numerical analysis method RK4 to model the vortex propulsion capabilities of four species of *Scyphozoan* jellyfish: moon jellyfish (*A. aurita*), Pacific sea nettles (*C.fuscescens*), cannonball jellyfish (*S. meleagris*), and lion's mane jellyfish (*C. capillata*). These jellyfish have similar anatomical structures, but represent a range of different sizes and unique bell shapes. They are also frequently studied and observed, meaning there is increased availability of photographic and videographic reference materials to inform the visualizations in the simulations. In each of their swim cycles, *Scyphozoan* jellyfish create a vortex ring in the water with their bell, which we model as two point vortices, one on either side of a cross section of the bell. The strength of these point vortices are adjusted to accommodate the varying speeds of different sizes of jellyfish, and in a given simulation the strengths of the vortices on either side of the bell can be selected to make the jellyfish turn during the video. The vorticity of each point vortex can be set to change after ten-second intervals, providing greater capability to replicate videos of live jellyfish swimming. We can ob-

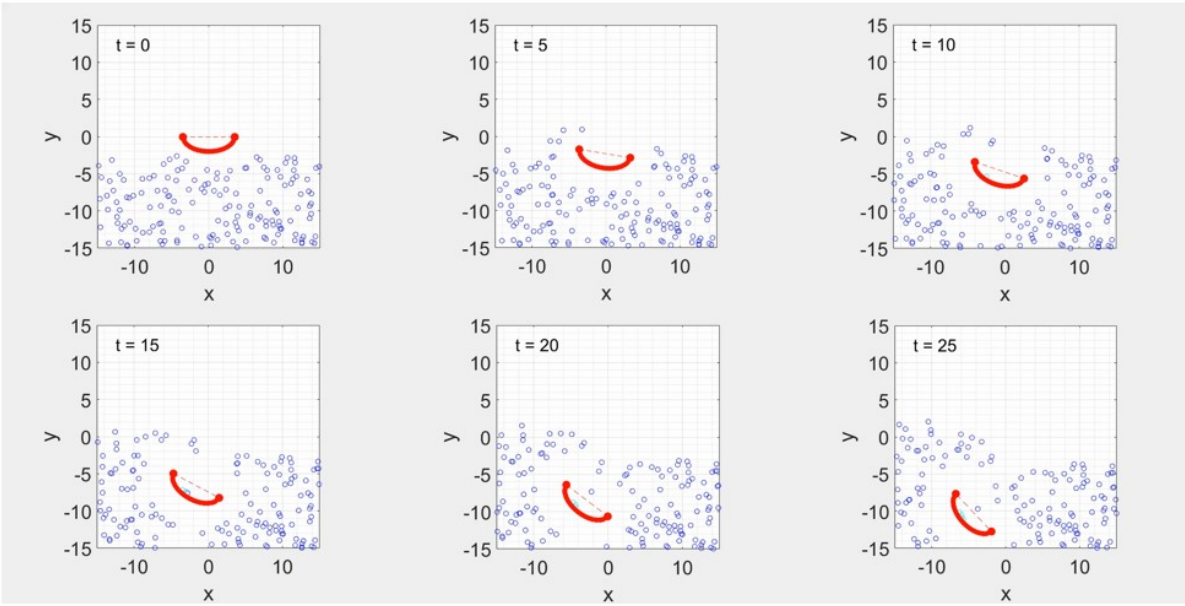

Figure 8: A simulation of a moon jellyfish swimming in an arc to the left with randomly distributed massless points in the vicinity. $\Gamma = [25, -15]$

serve the strength of the rotation of each of these point vortices directly by incorporating vectors into the background of the simulations.

All the selected species consume plankton as a part of their diet, which increases the importance of material transport in their vicinity as they swim. As a result, when we include particles in the simulation, which are approximated as massless, we can analyze their movement as if they are food for the jellyfish, meaning it can passively collect food as it swims. The points representing the massless particles are randomly initialized with a uniform distribution, and the distribution of the particles is not affected by the jellyfish species selected.

Developing this model will enable more applications in biomimetic engineering related to jellyfish. By mathematically describing the way jellyfish move, researchers can better design vehicles and robots to emulate this efficient system of movement. Examining jellyfish motion has the power to transform the way we design mechanisms for underwater travel, especially in situations in which we can prioritize efficiency and dependability over speed. By comparing models of a variety jellyfish of species, we will be able to determine which designs will be most effective for specific applications. These kinds of designs also have the capability to collect particulate matter from the environment of the jellyfish-modeled vehicle or robot. In order to inform these future engineering projects, the model and related videos produced in this project will persist as educational resources for understanding the vortex-powered swimming mechanism of jellyfish. With this model, future developers will have a better idea of what kind of bell shape is needed for their speed and particle collection applications. These designs will be able to effectively borrow from the naturally evolved efficiency of jellyfish, and the variety in shapes and sizes of species can potentially suit various engineering needs.

One area in which this project could be expanded is through the inclusion of the stopping vortices in addition to the starting vortices in the current model. The vortex-vortex interactions produced by the stopping and starting vortices contribute greatly to the efficiency of the jellyfish, so incorporating these into the model would be particularly helpful to fully understand the propulsive power of jellyfish, especially in physics and engineering contexts. The current model utilizes jet propulsion regardless of the oblate or prolate bell shape of the species, which is characteristic of faster-moving jellyfish. However, those jellyfish which use both stopping and starting vortices utilize rowing propulsion which leads to greater efficiency of motion. If the model is adapted to incorporate stopping vortices, and consequently rowing propulsion, it would enable a more extensive comparative analyses of various species.

Additionally, the code developed for this project could be expanded to include more parameters to generate simulations specifically to simulate particular videos of real jellyfish swimming. While several videos of jellyfish were replicated using the current model, jellyfish frequently change direction while swimming, meaning the vorticity values are required to change during the simulation. The code accommodates a sequence of three pairs of values to set the strength of the vortices on either side of the bell. If a longer video were being replicated, then a longer sequence could be required, but would not be difficult to incorporate. Similarly, the model could also benefit from the incorporation of parameters to represent characteristics of the jellyfish such as length of the pauses between contraction and relaxation of the bell, as well as characteristics of the surrounding environment, such as the density of the surrounding fluid, the drag produced from the fluid, or the size and mass of the particulate matter.

Overall, the model produced in this project increases the mathematical understanding of how jellyfish swim by creating simulations which demonstrate the swimming mechanism that contributes to their efficiency. The nature of their motion is such that they can passively move particulate matter from the surrounding environment into their bell. The results included above are a sample of the variety of simulations which can be produced using the model. The code is designed to provide many options for visualization, enabling the generation of simulations to suit many possible needs, including reproduction of documented motion of actual jellyfish, better physical understanding of how jellyfish move, and specific simulations to inform biomimetic design.

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