Optimal Minimal-Perturbation University Scheduling With Instructor Preferences

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Abstract - In the university scheduling problem, sometimes additions or cancellations of sections of a course occur shortly before the beginning of the academic term, necessitating last-minute teaching staffing changes. We present a decision-making framework that both minimizes the number of course swaps, which are inconvenient for instructors, and maximizes instructors' preferences for section times they wish to teach. The model is formulated as an integer linear program (ILP). Numerical simulations for a hypothetical mid-sized academic department are presented.

Keywords : scheduling; university scheduling; integer program; minimal perturbation

Mathematics Subject Classification (2020) **:** 90B06; 90B35; 90C11

1 Introduction

In this paper we present an integer linear program (ILP) model for reassignment of university teaching faculty to sections when a last-minute change is necessitated. We begin with a literature review of related work. We then derive and explain our ILP model. Finally, we conclude with several numerical simulations that confirm the validity of our model.

The problems of university timetabling (selection of times and rooms for each section) and scheduling (selection of instructors for each section) have long been a focus of study in the operations research community, dating back at least to the 1970s.[2, 11] A number of models for university course scheduling and timetabling have been proposed. The overall decision-making problem contains subproblems including (a) finding time slots for course sections, (b) deciding which section is assigned to a particular room at a given time, (c) deciding which instructor is assigned to a given section, and so forth.[3] In this work we focus on the dynamic reassignment problem that sometimes occurs after a schedule has been chosen, due to issues such as unexpected enrollment fluctuations or loss of an instructor. In this case, we wish to develop a new schedule which minimizes in some sense the number of course swaps, which are inconvenient for instructors due to the preparation time needed for each separate course; while at the same time taking into consideration instructors' preferences for times they wish to teach sections.

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In the literature, the reassignment problem is sometimes referred to as a "minimal perturbation" problem (MPP). Barták, Müller, Rudová, and Murray used MPP in the context of university course scheduling and have published several papers on the topic.[1, 8, 10] Their approach is called "iterative forward search" which operates over feasible, but incomplete solutions (meaning some variables are unassigned). Their work builds off of El Sakkout, Richards, and Wallace[4, 5] who introduced the MPP for general dynamic programs. More recent work in high school scheduling has been that of Kingston[6], who describes an algorithm called "polymorphic ejection chains," which repair an infeasibility while possibly creating a new one, in a successive fashion until the solution is feasible. Finally, Phillips, Walker, Ehrgott and Ryan[9] minimize course swaps in the university scheduling problem by only searching for solutions in a neighborhood of the original (now-infeasible) solution, and expanding the neighborhood until a solution is found.

In contrast to these works, the model we present here represents a new approach where swaps are minimized through explicit inclusion in the objective function. This approach has several benefits. First, ours is an exact solution method that does not rely on heuristics, so an optimal solution is guaranteed. Second, since both the minimizing of perturbations as well as a weighted sum of instructor time-preferences are considered in the objective function, we favor swaps that improve the times at which instructors teach. This also enables the decision-maker to choose the relative importance of these two objectives. Our formulation is an integer linear program (ILP) and thus can be solved with virtually any commonly-used optimization software package; there is no need for a problem-specific solution algorithm.

In this paper we borrow notation from a model put forth by Kumar.[7] That model is also an ILP that handles the assignment of sections to time slots and of instructors to sections. The objective function is a linear weighted sum of instructor time-preferences. Note that Kumar's model is for the original scheduling assignment problem and *not* the reassignment problem.

2 Model

Let I be the set of all course sections, J be the set of all instructors, and T be the set of all time slots. Let $\hat{i}$, $\hat{j}$, and $\hat{t}$ be the size of each set respectively.

Decision variables

- Let $P \in \mathbb{Z}^{\hat{i}\times\hat{j}\times\hat{t}}$, where

  $$P_{ijt} = \begin{cases} 1 & \text{if section } i \text{ is added to instructor } j \text{ in time slot } t \\ -1 & \text{if section } i \text{ is removed from instructor } j \text{ in time slot } t \\ 0 & \text{otherwise} \end{cases}$$

  for all $i \in I$, $j \in J$, $t \in T$.

- Let $T \in \mathbb{Z}^{\hat{i}\times\hat{j}}$. These decision variables will be necessary to linearize the objective function; see below.

Parameters

- Obsolete schedule: Let $X \in \mathbb{Z}^{\hat{i}\times\hat{j}\times\hat{t}}$.
  $X_{ijt} = \begin{cases} 1 & \text{if section } i \text{ had been assigned to instructor } j \text{ in time slot } t \\ 0 & \text{otherwise} \end{cases}$

  The new schedule is $X_{ijt} + P_{ijt}$.

- Instructor time-preference matrix: Let $W \in \mathbb{R}^{\hat{j}\times\hat{t}}$. $W_{jt} \geq 0$ indicate instructor j's preference for teaching in time slot t. Higher scores correspond to more desirable times. 0 indicates unavailability during time slot.

- Instructor time availability matrix: Let $F \in \mathbb{Z}^{\hat{j}\times\hat{t}}$.
  $F_{jt} = \begin{cases} 1 & \text{if } W_{jt} > 0 \\ 0 & \text{if } W_{jt} = 0 \end{cases}$

  $F_{jt} = 1$ indicates that instructor j is available in time slot t, and $F_{jt} = 0$ otherwise.

- Course swap penalty matrix: Let $\alpha \in \mathbb{R}^{\hat{i}\times\hat{j}}$. $\alpha_{ij} \geq 0$ are penalty factors for making changes to section i in instructor j's assignments; see below.

  This penalty applies for either adding a section that was previously not in instructor j's course assignments, or for removing a section that was previously in instructor j's assignment. Simply changing to a different section of an existing course incurs no penalty; however, note that a reward/penalty for changing the time slot to a more or less desirable time for instructor j is handled via W_{jt}. Our reasoning for this modeling decision is that while instructors are inconvenienced by the changing of section times, they would be additionally inconvenienced by gaining a section of a new course or losing the last section of a course, as presumably they have already prepared their course materials at the time this reassignment is decided. Due to this decision, new assignments may be significantly different from obsolete assignments if identical or near-identical values for many entries in W_{jt} are assumed; this could be mitigated through inclusion of a third objective function term that minimizes such section swaps. We defer such analysis to future work.

- Section-time slot matrix: Let $M \in \mathbb{Z}^{\hat{i}\times\hat{t}}$. M_{it} = number of sections of section i scheduled at time slot t. Note that while much of our notation was borrowed from Kumar's model, our definition of M_{it} differs.

- Instructor-section matrix: Let $C \in \mathbb{Z}^{\hat{i}\times\hat{j}}$.
  $C_{ij} = \begin{cases} 1 & \text{if section } i \text{ can be taught by instructor } j \\ 0 & \text{otherwise} \end{cases}$

- Teaching load: Let $N^+ \in \mathbb{Z}^{\hat{j}}$, $N^- \in \mathbb{Z}^{\hat{j}}$, and $H \in \mathbb{Z}^{\hat{i}}$. N_j^+ (N_j^-) is maximum (minimum) number of sections/credit hours that can be assigned to instructor j. H_i = number of sections/credit hours that are assigned to section i.

At some institutions, teaching load is determined by number of sections; in others, it is determined by credit hours. Furthermore some instructors (often part-time) may contractually have a range on how many sections or credit hours they teach, whereas other instructors may have a fixed number of sections or credit hours. Our model is flexible enough to allow for all such possibilities. If instructor j must teach an exact number of sections or credit hours, then $N_j^+ = N_j^-$. Zero values are allowed to allow for faculty to possibly teach no section.

Objective function

The objective function is the weighted sum of instructor time-preferences, plus the weighted sum of the absolute value of sections added and subtracted:

$$\max \underbrace{\sum_{j\in J}\sum_{t\in T}\left(W_{jt}\sum_{i\in I}P_{ijt}\right)}_{\text{instructor time preferences}} - \underbrace{\sum_{i\in I}\sum_{j\in J}\left(\alpha_{ij}\left|\sum_{t\in T}P_{ijt}\right|\right)}_{\text{course swaps}}$$

Note that this objective function is not linear but can be made linear through the introduction of the new variables T_{ij}. We arrive at our final form:

$$\boxed{\max \sum_{j\in J}\sum_{t\in T}\left(W_{jt}\sum_{i\in I}P_{ijt}\right) - \sum_{i\in I}\sum_{j\in J}\alpha_{ij}T_{ij}}$$

along with additional constraints labeled "new variable constraints" below.

Constraints

- New variable constraints:

  $$T_{ij} \geq \sum_{t\in T} P_{ijt} \quad \forall i \in I, \quad \forall j \in J$$

  $$T_{ij} \geq -\sum_{t\in T} P_{ijt} \quad \forall i \in I, \quad \forall j \in J$$

  These constraints are necessary to convert the $|\sum_t P_{ijt}|$ term in the objective function into linear form.

- New schedule is binary:

  $$X_{ijt} + P_{ijt} \geq 0 \quad \forall i \in I, \quad \forall j \in J, \quad \forall t \in T$$

  $$X_{ijt} + P_{ijt} \leq 1 \quad \forall i \in I, \quad \forall j \in J, \quad \forall t \in T$$

  This set of constraints ensures that you cannot add a section to which an instructor already has been assigned, nor can you remove a section from which an instructor was not previously assigned.

- Assign all sections:

  $$\sum_{j\in J}(X_{ijt}+P_{ijt}) = M_{it} \quad \forall t \in T, \quad \forall i \in I$$

  These constraints ensure that every section of every course is assigned to an instructor.

  $$\sum_{t\in T}(X_{ijt}+P_{ijt}) \le C_{ij}\sum_{t\in T} M_{it} \quad \forall i \in I, \quad \forall j \in J$$

  These constraints ensure that instructors only teach courses they are able to teach.

- Instructors teach only during their available times:

  $$\sum_{i\in I}(X_{ijt}+P_{ijt}) \le F_{jt} \quad \forall j \in J, \quad \forall t \in T$$

  These constraints ensure that instructors are not assigned during times they are not available, and also that an instructor may teach no more than one section simultaneously.

- Teaching load requirements:

  $$\sum_{i\in I} H_i \sum_{t\in T}(X_{ijt}+P_{ijt}) \le N_j^+ \quad \forall j \in J$$

  $$\sum_{i\in I} H_i \sum_{t\in T}(X_{ijt}+P_{ijt}) \ge N_j^- \quad \forall j \in J$$

  These constraints ensure that each instructor teaches the correct number of sections.

- Avoid time slot conflicts: Let Γ be the set of all pairs (t', t''), where $t' \in T$ and $t'' \in T$, and $t' \neq t''$, such that times t' and t'' conflict.

  $$\sum_{i\in I}(X_{ijt'}+P_{ijt'}) + \sum_{i\in I}(X_{ijt''}+P_{ijt''}) \le 1 \quad \forall j \in J, \quad \forall (t', t'') \in \Gamma$$

  These constraints prevent an instructor from teaching at both time slots t' and t''. Perhaps t' and t'' overlap or perhaps we wish to prevent instructors from teaching *both* early mornings and late evenings, or *both* Mon/Wed/Fri and Tue/Thur, etc.

- Decision variables are binary:

  $$-1 \le P_{ijt} \le 1 \quad \forall i \in I, \quad \forall j \in J, \quad \forall t \in T$$

  $0 \le T_{ij} \le 1 \quad \forall i \in I, \quad \forall j \in J$ This enforces that each element of the decision variable $P_{ijt} \in \{-1, 0, 1\}$ and $T_{ij} \in \{0, 1\}$.

3 Simulations

We now present numerical simulations for a hypothetical department offering 57 sections of 17 different courses, with 13 full-time and 9 part-time instructors, during 24 possible time slots. These values were loosely based on the Fall 2020 semester schedule for the Mathematics Department at the University of Portland.

All full-time instructors must teach 3 sections while part-time instructors may teach between 0 and 2 sections. Some pairs of time slots conflict with each other. We assume for simplicity that the W_{jt} and α_{ij} matrices are all 1s, indicating that all instructors are equally satisfied teaching at any time and are equally inconvenienced by adding or dropping a section. M_{it} represent actual sections of courses for Fall 2020 at the University of Portland. For C_{ij}, an attempt was made to group together instructors based on their teaching area specialties; for example, part-time instructors can teach any lower-division course, while pure mathematicians can teach lower-division courses plus pure upper-division courses, whereas applied mathematicians can teach lower-division courses plus applied upper-division courses. X_{ijt} are a simulated feasible solution based on these constraints.

Note that X_{ijt} do not represent the actual schedule executed by the University of Portland as that schedule is sub-optimal according to our assumptions on the parameter values, and we wish to begin with an optimal solution for X_{ijt}. Another possible path would be to use techniques of inverse optimization to find what parameter values could have resulted in the schedule actually implemented at the University of Portland, and assigned this to X_{ijt}. We leave this as a direction for future work.

We examine several simulations, each of which represents small perturbations to a system with easily understood parameter choices, so that the optimal solution is known. This provides numerical evidence that the model behaves as expected.

The ILP involved 9350 variables and 39238 constraints. The problem was solved in Matlab R2016b, using the "intlinprog" function in the Optimization Toolbox. Elapsed time to run the entire code, including reading in data from a spreadsheet, setting up constraints to pass to the solver, and displaying the output, was approximately 4 seconds on a single 2.8 GHz Intel Core i5 processor for each simulation that follows; the time for just the solution of the ILP was approximately 2 seconds.

Simulation 1: Removal from part-time instructor

We remove one section of one course that is taught by part-time instructor "PT1" in X_{ijt}. As expected, $P_{ijt} = -1$ for that section-instructor-time slot combination, and $P_{ijt} = 0$ for all other combinations indicating that no other changes occur.

Simulation 2: Removal from full-time instructor

We remove one section of one course that is taught by full-time instructor "FT1" in X_{ijt}. Because full-time instructors must teach 3 sections, they are reassigned a section that had been taught by part-time instructor PT1. Thus we have $P_{ijt} = -1$ in two places (the removed section and the section lost by the part-time instructor) and $P_{ijt} = 1$ in one place (the section moved from part-time to full-time), and $P_{ijt} = 0$ elsewhere. We further note that the full-time instructor is swapped to another section of the same course, which results in $\alpha_{ij} = 0$.

Simulation 3: A larger perturbation

In both previous simulations, our model was able to adjust so that no instructor was forced to take on a new course, as opposed to switching sections of an existing course; in other words, the course swap penalty α_{ij} was never activated (i.e., $T_{ij} > 0$ for some i, j.). However, if we push the system far enough, this becomes inevitable. For example, suppose 8 sections of a course, "course 2," are offered initially, but for some reason 4 of those must be eliminated. Of the 4 eliminated, 3 had been assigned to part-time instructors PT2, PT3, and PT4; and 1 section assigned to full-time instructor FT1. This represents a more drastic last-minute scenario than only one section being removed, as had been the case in simulations 1 and 2.

The solution to our ILP contains several components. First, the sections of course 2 assigned to PT2, PT3, and PT4 are simply eliminated, as expected. The more interesting result is that FT1 is shifted to a section of a completely different course, "course 1," meaning that $T_{ij} > 0$ for some i, j. The previous instructor of that section of course 1, part-timer PT1, loses the course.

This simulation presents a more interesting situation as the decision of which section(s) to remove from part-time faculty immediately before the academic term is often fraught, understandably for those instructors who were expecting to teach. One can imagine that PT1 would be surprised to lose their section despite the fact that it was course 2, and not course 1, that had the cancelled sections. While such decisions will always be difficult, our model offers a way of choosing which such section(s) to remove in an optimal way.

A summary of simulations 1-3 can be found in table 1.

4 Conclusion

We have developed a decision-making framework for last-minute teaching staffing changes when sections are added or removed for an academic department. This framework minimizes swaps in such a way that it penalizes changing an instructor to a different course but not a different section of the same course. It also maximizes instructors' preferences for times they wish to teach. The relative importance of these factors to each other as well as across instructors are handled through the weight matrices W_{jt} and α_{ij}. Our framework is easily and quickly solvable using off-the-shelf ILP solvers. Last-minute schedule changes are stressful to all involved; our hope is that this framework provides a tool that decision-makers can use to quickly, optimally, and objectively reshuffle teaching assignments when necessary. While our model does not take into account every factor in determining a schedule (issues such as seniority, for example, are not considered), it can provide a starting point for intra-departmental conversations.

In our formulation, our objective function was optimized with known constraints for a given set of parameters α_{ij}. The form of the objective function is additive in the two simultaneous goals of minimizing course swaps and maximizing instructor time preferences. Therefore α_{ij} can be equivalently thought of as a weighing factor which describes the relative importance of these two conflicting goals. One avenue for future work is to reformulate our model in a multiobjective framework. This could be achieved by performing a sensitivity analysis on α_{ij} to obtain an optimal set of non-dominated solutions.

Another question that arises from the simulations is how to choose reasonable values for the parameters. Future work could include using inverse optimization techniques to reverse-engineer parameter values by considering actual schedule changes made by the decision-maker in previous instruction terms.

Simulation 1.

Sections removed			Sections added		
Course	Instructor	Section	Course	Instructor	Section
1	PT1	A	(None)		

Simulation 2.

Sections removed			Sections added		
Course	Instructor	Section	Course	Instructor	Section
1	PT1	A	1	FT1	A
1	FT1	B			

Simulation 3.

Sections removed			Sections added		
Course	Instructor	Section	Course	Instructor	Section
1	PT1	A	1	FT1	A
2	FT1	A			
2	PT2	B			
2	PT3	C			
2	PT4	D			

Table 1: Simulations of sections removed. Part-time instructors are numbered PT1, PT2, ... whereas full-time are numbered FT1, FT2, ... In simulation 1, section A of course 1 removed from PT1. In simulation 2, sections A and B of course 1 do not conflict. Section B of course 1 is removed from FT1, so a swap occurs of section A of course 1 from PT1 to FT1. In simulation 3, course 1 section A conflicts with course 2 section C, and sections A and B of course 2 conflict with each other. 4 sections of course 2 are removed, one of which had been assigned to FT1, so FT1 is reassigned from course 2 section A to course 1 section A.

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