

# An Examination of *Algebra for All* through Historic Context and Statewide Assessment Data

Jian-Hua Liang, Ed.D.  
California Department of Education

Paul E. Heckman, Ph.D.  
University of California, Davis

## ABSTRACT

Since 2003, California has enacted a policy through its education accountability system that encourages schools and districts to place all 8th grade students into algebra courses and therefore, be tested in algebra in the statewide assessment program. Ten years later, there are a great many more 8th graders taking algebra now. However, there are also many students repeating algebra, instead of going on taking higher level mathematics tests. This article aims to provide the historic context of this policy, previous and recent studies on 8th grade algebra, and our study based on the California Standardized Testing and Reporting (STAR) data. We analyzed 8th grade algebra test-taking and the following years' higher level mathematics test-taking to examine the college preparation course taking pipeline. Our longitudinal study compared two groups of students' performance on 9th grade algebra between those who previously scored below proficient on algebra at 8th grade and those who scored proficient or above on general mathematics at

8th grade. Further, another longitudinal study linked 7th grade mathematics sub-scores to 8th grade algebra achievement. The results show that "algebra for all" policy increased the number of students taking algebra at 8th grade and subsequently, taking higher level mathematics tests. However, the pipeline of the college preparation course taking has a significant leak because the number of students taking higher level mathematics decreased dramatically after algebra. Longitudinal study shows that students who pass the general mathematics test at 8th grade have a 69% greater chance to pass the algebra test at 9th grade compared to their peers who failed the algebra test at 8th grade. We also find that the sub-score rational numbers is a strong predictor of 8th grade algebra achievement. Alternatives to help all students achieve in mathematics learning are also discussed in addition to recommendations for future research.

## Historical Context of the *Algebra for All* Policy

Algebra has fit into the edifice of mathematics education in American secondary schooling in various ways for over a century. However, the focus on algebra in the stream of mathematics' curriculum reforms during the 20th century, and, thus, in today's 21st century, represent struggles about standards, including

questions about who developed the standards and the focuses of the standards, the fluctuating influence of mathematicians, and views of students and their future prospects. In particular, these struggles involve Herbert Spencer's question, what knowledge is of most worth? The struggles also extend to questions which students various reforms in mathematics will benefit: all, or only

those who are to go onto college? And, finally, do the presumed benefits accrue to the students for whom the reforms are intended to benefit? Understanding these struggles in historical context and being clear that it is the enacted thoughts and actions behind the classroom doors that will matter the most if students are to benefit provide an important backdrop for the data we analyze in this study to make a difference.

During the past 100 plus years, several critical periods of curriculum reform have brought attention to different focuses for mathematics in the school curriculum. One of those periods occurred at the turn of the 20th Century. Then, like now, concerns about immigration, dramatic developments in the economy and industry, the influence of the subject disciplines, such as mathematics, and the educational expectations held for different students attending schools arose. These concerns influenced the creation of commissions and study groups during the first two decades of the twentieth century. The commissions considered these factors and developed curricular reforms to respond to those concerns at that time. For example, the secondary school curriculum had special relevance as reflected in the work of the Committee of Ten, which began its work in 1892. It was comprised of university presidents and a faculty member, principals, and the United States Commissioner of Education. It focused on developing a common curriculum for all students who would attend secondary schools, not just the college bound students. As Cremin (1955) notes:

... its conception of the secondary school is here all-important. The secondary school is viewed as an institution designed to prepare a small segment of American youth "for the duties of life" by improving their intellectual abilities. The Committee saw absolutely no conflict between this conception and that of the high school as a college-preparatory institution, for the task of improving intellectual abilities centered squarely in the studies of the college (p. 296).

While Cremin points to the Committee of Ten's reform in the secondary schools at that time as one about influencing only a small segment of students

who would be going onto college, there was an indirect expectation that the reform would also influence the intellectual abilities of all students. That twin hopes have remained with us since that time. Nonetheless, algebra has remained a focus for the small group of college bound students. For example, in 1895, the National Education Association's Committee on College Entrance Requirements recommended algebra as a 9th grade course (George, 2007). However, since then, during the past century, and now during the first decade and more of the 21st century, efforts have fluctuated between having students focus on the knowledge and skills related to college preparation, like algebra, and those that would relate directly to student's preparation for life, which in turn would advance their knowledge and skills as citizens and in various occupations and careers that students might pursue after high school. For example, despite that early effort in 1892 to have algebra as a course for all students, schools developed programs in which students' focuses in mathematics shifted to more practical matters, in courses like business math, applied math, and others related to arithmetic content for students who were not on a college preparation track. Algebra "for all" was often dropped and replaced by courses such as general mathematics for students who were not directly college bound (Tyack & Cuban, 1995). As a result, in the 1920s, participation in algebra had dropped from 57% of students taking algebra in 1910 to only 40% taking algebra in 1922 in Ohio, for example (Stanic & Kilpatrick, 1992).

Other subsequent curriculum reform efforts sought more dramatic changes in the secondary school curriculum after that time. For example, in the 1930s and into the 1940s, the Progressive Education Association undertook the Eight-Year Study. It involved 30 high schools in advancing progressive educational practices. In their study efforts, those schools addressed the following concerns about high schools: the lack of purpose, limited attention to citizenship and community life, the lack of intellectual challenge, weak personal connections to students, and ineffective classroom learning conditions (Aiken, 1942).

The remedies that arose in these 30 schools to address these issues can be summed in this way:

The schools [those involved] were confident

that this could be done, ... by basing the secondary school curriculum upon the needs of the youth in our society. If the high school helped students to find the meanings of their life experiences, they would go on to college to seek deeper and broader meaning in their maturing experiences. To the end, traditional studies would have to be revitalized and re-oriented: much new content would have to be included in the curriculum of school and college. ... These schools took their eyes off of the college gates and looked to the fruitful fields beyond (p. 23).

Here again, another effort to have a school curriculum do both – have students prepared for college and have a productive life beyond schooling.

Despite the success of the schools' efforts involved in the Eight-Year Study (Chamberlin, Chamberlin, Drought, & Scott, 1942), eventually, national attention then turned to advancing "Life Adjustment" goals, related to and different in degree from the efforts in the 30 schools of the Eight-Year Study. These reforms emphasized, "active and creative achievements as well as an adjustment to existing conditions; it places a high premium upon learning to make wise choices, since the very concept of American democracy demands the appropriate revising of aims and the means of attaining them" (Cremin, 1964, p. 336). While the Life Adjustment curriculum succeeded in gaining attention and implementation in many schools in the country, it, too, waned under predictable criticisms revolving around the abandonment of conventional subjects and courses (Cremin, 1964, p. 339). Yet, as before, algebra and similar courses prevailed for those going to college. For those not going on to college, the debate continued. On what should these students focus with respect to mathematics in high school?

For example, in 1957, Sputnik was launched into the sky, and on the horizon a new focus for mathematics reform – an update to the mathematics and science curriculum so that the United States would overtake the perceived technological superiority of the Soviet Union. Yet, even before the launch of Sputnik, during the 1950s, university mathematicians worked again to rethink school mathematics and involve themselves in

school mathematics curriculum reform. Max Beberman, for example, created the New Math, with his colleagues on the University of Illinois Committee on School Mathematics (UICSM). Like the School Mathematics Study Group (SMSG) that followed UICSM, it focused on algebra and the integration of concepts like "structure and proof in algebra", "treatment of inequalities along with equations", and "integrated algebra and trigonometry" into the school mathematics program (Herrera & Owens, 2001). These refocuses would create a pathway to college and college preparation as well as success in mathematics for other students to benefit them beyond schooling (Stanic & Kilpatrick, 1992).

University mathematicians in particular once again entered the arena of curriculum reform, attempting to bridge the gap between the existing school mathematics' curriculum and the discipline of mathematics underway in universities. Attention to the interests of university mathematicians moved reform in mathematics farther away from methods of learning and the centrality of students, as expressed in the curriculum being advanced in the Eight-Year Study and the Life Adjustment movements during the 1930s and 1940s. It again re-emphasized mathematics' content and courses. Stanic and Kilpatrick (1992) explains this rejection of the earlier curriculum changes and the responses to Sputnik in this way:

... a previous overemphasis on method was by many held responsible for the neglect of content because the university mathematicians who dominated the modern mathematics movement tended to be specialists in pure rather than applied mathematics, they saw pure mathematics, with an emphasis on set theory and axiomatics, not only as the content that was missing from the school curriculum, but also as providing the framework around which to reorganize that curriculum (p. 412).

This refocus on the discipline of mathematics did not persist for too long or affect much of life behind classroom doors in mathematics for students or teachers (Goodlad, Klein, & Associates, 1970). The advocacy for basic skills for most students had returned this time

in the form of competency-based education (Cooney, 1988). By the 1970s, the new curriculum manifestations of the discipline of mathematics focused only on the gifted and advanced students. For all other students, general mathematics and basic skills remained the menu of the day. Tracking was fully embraced with differentiated educational goals and curriculum being enacted for students with different backgrounds and experience (Oakes, 1985; Oakes, 1990; Oakes and Guiton, 1995; Ravitch, 2000).

After the mid-1980s, *A Nation at Risk* became the policy reform text of the day, despite consisting of fewer than 25 pages of analysis and recommendations. Nevertheless, it called for educational reforms for achieving a more competitive stance towards other nations, similar to the responses that arose in the late 50's to the orbiting Sputnik satellite (The National Commission on Excellence in Education, 1983). Mathematicians, again, entered the educational reform scene, emphasizing this time the ways of knowing in the academic discipline of mathematics and adding attention to insights from cognitive sciences for a more powerful kind of mathematics learning for all students.

In 1989, the National Council of Teachers of Mathematics (NCTM), as a result of the "standards movement," which grew out of *A Nation at Risk*, decided to use its own organizational resources to draft a set of standards determined by the profession – mathematics educators in P-12, university level mathematicians, teacher educators, and practicing teachers to name a few of the interests involved. As a result of their work, they produced an enumeration of standards for practice. At least two other documents articulating related standards for teaching and standards for assessment emerged in 1991 and 1995 (Hiebert, 1999).

These efforts established a set of goals for mathematics education. The goals centered on several emotional and cognitive aspects of learning in mathematics – valuing mathematics and having confidence in undertaking mathematical thinking and problem solving. The remaining goals centered on the qualities of problem solving and reasoning as mathematicians accompanied with the skill of communicating clearly as a mathematician about these qualities. Romberg (1992) notes the intended focus of classroom learning for students:

...encourage them to value the mathematical enterprise, to develop mathematical habits of mind, and to understand and appreciate the role of mathematics in human affairs; that they should be encouraged to explore, to guess, and even to make and correct errors so that they gain confidence in their ability to solve complex problems; that they should read, write, and discuss mathematics; and that they should conjecture, test, and build arguments about a conjecture's validity. The opportunity for all students to experience these components of mathematical training is at the heart of our vision of a quality mathematics program (p. 424).

Like other mathematics' curriculum reform efforts during the century, this one, too, sought these focuses in order to accomplish the twin goals of preparing students for college and advancing student success beyond their school years. As before, critical reactions followed. The evidence suggested that having these goals enacted in classrooms was not a foregone conclusion (Herrera & Owens, 2001). But having all students attend to a mathematics that would do both – advance college preparation and life preparation – followed as the 20th century came to a close.

For example, in 1994, the New York City public school system decided that all of its 9th grade students would take algebra as part of a college preparation curriculum (Bradley, 1994). The Chicago public schools followed in 1997 with a similar requirement (Viadero, 2009). Other national groups have also agreed with this emphasis on college preparation mathematics and science. For example, the College Board's "Equity 2000" program expands algebra courses to high school freshmen nationwide (2000). After 2000, when the California legislature passed a bill (Senate Bill 1354) requiring high school candidates for graduation to successfully complete an algebra course (*California Education Code*, Section 51224.5), algebra became a required course for all California high school graduation candidates.

Algebra is now widely considered to be a gateway course for college preparation (Riley, 1997; Moses,

Kamii, Swap, & Howard, 1989). In the “A-G” subject requirements for admission to the University of California (UC) and the California State University (CSU), algebra is designated as the first of a sequence of three courses (algebra I, geometry, and algebra II) deemed necessary for college preparation (University of California, 2007).

### **Algebra for All as a Civil Right and a State Policy**

In the last three decades, all over the United States, “algebra for all” has become a mantra in the movement to disrupt the tracking system, and advance the twin goals that we have been discussing. Early in 1987, Civil Rights crusader Robert Moses took the notion of “algebra for all” to the 7th grade through his Algebra Project in Cambridge, Massachusetts. He worked with an inner-city school community, modeling his thinking on some of the successes he had in the Civil Rights Movement in Mississippi. Specifically, he urged parents to enroll minority students in algebra courses in 7th grade, recommended that teachers develop a curriculum that makes algebra more relevant for the students, and encouraged students to believe that achievement resulted from hard work rather than innate ability (Moses, Kamii, Swap, & Howard, 1989).

The success of the Algebra Project inspired many educators nationwide to move more algebra courses into grades 7, 8, and 9 classrooms; and students took these courses in increasing numbers. However, it was the publication of *Mathematics Equals Opportunity* (Riley, 1997) that turned 8th grade algebra into a policy issue in education reform. The white paper prepared by then U.S. Secretary of Education Richard Riley reported that students who “begin to study algebra during middle school are at a clear advantage of going to college and completing college” (p. 16).

Many studies using the National Center for Educational Statistics’ (NCES) National Education Longitudinal Study (NELS: 88) data (McLaughlin, Cohen, & Lee, 1997) concluded that, by taking algebra in 8th- grade, students had a greater chance of going to college (Spielhagen, 2006; Smith, 1996; Stevenson, Schille, & Schneider, 1994). Smith (1996) concluded that “early access to algebra had a sustained positive effect on students, leading to more exposure to advanced mathematics curriculum and, in turn, encouraging

higher mathematics performance by the end of high school” (p. 148). Spielhagen (2006) found that “students who completed algebra in 8th grade stayed in the mathematics pipeline longer and attended college at greater rates than those who did not” (p. 35). He suggested greater access to algebra in 8th grade as a means of closing the achievement gap in math.

Although California’s public school curriculum frameworks did not specify algebra as an 8th grade mathematics standard, the 2003 California assessment and accountability system deemed algebra to be an end-of-course (EOC) test for students in grades 8 through 11. The state’s education accountability system penalizes schools and districts for not testing 8th- and 9th-grade students in algebra or higher mathematics EOC. Since then, the percentage of 8th-grade students taking algebra has risen from 34% in 2003 to 59% in 2011 (California Department of Education, 2003; 2011). Also, since 2006, when California started allowing 7th-grade students to take the California Standards Test (CST) for Algebra I, the number of students in grade 7 taking the algebra CST has increased from 22,000 in 2007 to 38,000 in 2011, which is 4.4% and 8.1% of the state’s 7th-grade population, respectively (California Department of Education, 2007; 2011). On July 9, 2008, the California State Board of Education (SBE) decided that within three years all schools would be assessing their 8th-grade students in algebra.<sup>1</sup>

### **Unintended Consequences of the Policy for Student Achievement**

These nationwide and statewide educational endeavors during the past several decades have greatly increased the numbers of students enrolling and succeeding in algebra classes (Bozick & Owings, 2008). However, this unprecedented access to algebra courses has brought with it a widened spectrum in learning achievement. Test results led to the charge that enrolling more students in algebra classes did

1 After the SBE’s decision, the Association of California School Administrators (ACSA) and the California School Boards Association (CSBA) filed a law suit against the SBE on the decision. The plaintiffs later were joined by the California Teachers Association (CTA) and Superintendent of Public Instruction Jack O’Connell. On December 19, 2008, a superior court judge issued a preliminary injunction which required the SBE to cease action associated with its July vote to mandate all eighth graders take the Algebra I California Standards Test.



not improve California students' overall mathematics achievement (Center for the Future of Teaching & Learning, 2005). Many educators speculated that placing more and more 8th-grade students into algebra courses would only increase the failure rate, as reflected in scoring *proficient* or above on the CST for Algebra I, and, then, inevitably increase the number of students repeating algebra in 9th grade. Students who repeated algebra would do a lot worse than those students who take algebra for the first time.

As shown in the state's Superintendent of Public Instruction's test results release in 2012, only 25% of students scored *proficient* or above in repeating the CST for Algebra I compared to 38% of students, who took the CST for Algebra I for the first time (Torlakson, 2012, August 15, Table 6). In 2007, 44% of California 9th-grade students took algebra over again after first taking it in 8th grade, largely due to their previous year's below-proficient scores on the CST for Algebra I (Liang & Guo, 2007). According to a recent report by the Brookings Institute, 120,000 8th-grade students nationwide have been misplaced in algebra classrooms (Loveless, 2008). Loveless found that some of the misplaced students were functioning about 7 grade levels below peers enrolled in the same courses according to NAEP scale scores (p. 7).

### From Access to Outcome: Asking Hard Questions

Policy makers often see "algebra for all" as a way to address the equity issue for students of minority and low-income families. Allowing these students access to college preparatory courses democratizes 8th-grade algebra and promotes social justice (Loveless, 2008, p. 3). However, as accountability has shifted from access to outcomes, focusing on student achievement to determine the policy's effectiveness is a matter of increasing concern (Shullock & Moore, 2007). As the No Child Left Behind (NCLB) law aims at closing the achievement gap between minority and poor students on the one hand and white and middle class students on the other, educators and policy makers are facing a tough question: Is it appropriate to place in algebra courses those 8th-grade students, who appear to be not prepared and likely to fail, considering that there are not sufficient resources and effective academic

support to advance their success?<sup>2</sup>

Of course, there is also the issue of whether the "algebra for all" policy translates into teacher action and classroom practices reflecting the intent of the policy. W.W. Charters, Jr. and John E. Jones (1973) raised a concern about the risk of appraising non-events in program evaluation because implementations of policy could be "more fictional than factual" (p. 5). For example, remedial mathematics courses have not disappeared from those same districts, where the "algebra for all" policy has been embraced, even as the trend to push algebra into 8th-grade classrooms has become noticeable (National Center for Education Statistics, 2005). According to the California Basic Education Data System (CBEDS)<sup>3</sup>, for example, remedial math still exists in many classrooms of the numerous California school districts that adopted "algebra for all 8th graders."

While a great many teachers and students are making enormous efforts to teach and learn algebra in 8th-grade classrooms, the staggering performance of these students on the statewide assessment forces educators to ask the hard question, whether providing the access to those students that algebra was a far reaching subject actually benefits them if they are not achieving the learning success. Educators must make tough choices to balance students' access and learning outcomes as schools and districts in California are under increasing pressures of raising students' test scores to meet the state and federal accountability requirements.

In this new climate, policy makers and educators are confronted with many other questions related to this "algebra for all" policy. Several studies based on California's Standardized Testing and Reporting (STAR) program from the state's middle schools and high schools shed light on the effect of increasing 8th-grade algebra test-taking. Kriegler and Lee (2006) studied over 100 middle schools in Southern California. They found that placing students who scored below basic

2 In August, 2009, California then Superintendent Jack O'Connell proposed the "California Algebra I Success Initiative," which called for \$3.1 billion to build the infrastructure for California schools to prepare all California 8th graders to succeed in algebra. Yet, this initiative was never funded.

3 A database collected by the California Department of Education. The online access is at this link: <http://www.cde.ca.gov/ds/sd/cb>

or far below basic in mathematical proficiency on their 7th-grade CST in 8th-grade algebra courses is ineffective because a large number of these students failed the CST for Algebra I. Their study concludes that a *proficient* or above score on the 7th-grade CST for Mathematics is a strong indicator for 8th-grade algebra success (p. 10). Waterman (2010) investigated 8th- and 9th-grade mathematics classes in eight school districts in Northern California's Bay Area. He found that many students repeated algebra and that repeating did not yield better results in 9th grade. A more recent comprehensive study on middle grade mathematics performance by Williams, Haertl, Kirst, Rosin, and Perry (2011) concluded that "placing all 8th graders into Algebra I, regardless of their preparation, sets up many students to fail" (p. 3).

Outside of California, Allensworth et al. (2009) studied data from Chicago schools. They found placing all 9th graders in algebra had few benefits. They concluded, "Although more students completed 9th grade with credits in algebra..., failure rates increased, grades slightly declined, test scores did not improve, and students were no more likely to enter college" (p. 367). Clotfelter and his colleagues (2012) reported on the negative impact of 8th-grade algebra in the Charlotte-Mecklenburg schools in North Carolina. They noted that, "students affected by the acceleration initiative scored significantly lower on end-of-course tests in Algebra I, and were either no more likely or significantly less likely to pass standard follow-up courses, Geometry and Algebra II, on a college-preparatory timetable" (p. 1). Finally, as noted earlier, in *The Misplaced Math Student: Lost in Eighth-Grade Algebra*, Loveless (2008) found that 120,000 students nationwide were misplaced in 8th-grade algebra classrooms.

### Two Elements of Algebra Success: Placement and Preparation

This paper then aims to provide empirical evidence in two critical elements of algebra success for all: student placement and algebra preparation. We make use of our most recent studies to answer three questions (Liang, 2009; Liang, Heckman, & Abedi, 2012). First, regarding both placement and preparation, we looked at whether the increase in the number of California 8th-grade students taking algebra has achieved the goal

of the "algebra for all" policy, namely, increasing the overall college preparation course-taking pipeline. We discovered that the answer is yes, but this pipeline has a significant leak in it. Second, regarding preparation, we asked what are the differences in both 9th-grade test taking rate and performance between (a) those students who took the CST for Algebra I at 8th grade and scored below *proficient*, and (b) those students who took the CST for general mathematics at 8th grade and scored *proficient* or above? We discovered that students who are proficient in General Mathematics at the end of 8th grade do better in algebra in high school than do those students who take algebra but do not do well. Third, regarding placement, we wanted to know what subset of the content domain of mathematics, its knowledge, and skills account for 8th-graders' algebra scores. Our hope was to provide a means whereby educators could more accurately place 8th-grade students in the level of course more likely to benefit their mathematical development.

### Method

The data sources for this investigation are the students' test results from the California Standardized Testing and Reporting (STAR) program. We used two data sources: one is the aggregate data reported by the California Department of Education from 2003 to 2011<sup>4</sup>; the other is the STAR student level data files administered in 2006 and 2007. These data files were obtained from the California Department of Education in November, 2007. Several factors guided us to choose these two years for the study. First, year 2006 is the last year that all 7th-grade students take the CST for Grade 7 Mathematics. Starting 2007, students in grade 7 who took algebra courses are allowed to take the CST for Algebra I, instead of CST for Grade 7 Mathematics. Second, after 2007, the California Modified Assessment (CMA) was developed for students with disabilities who have an individualized education program (IEP) and meet the criteria for taking the CMA. With the CMA, many students who would have taken the CST are no longer in the data files. Last but not the least, the use of Statewide Student Identifier (SSID) was first implemented in 2006, though on a voluntary basis, with

4 These data can be retrieved at <http://www.star.cde.ca.gov>

a participation rate (over 95%) sufficient for this study. The SSID, then, became mandatory in 2007. The SSID enables researchers to conduct longitudinal studies by matching student records from year to year.

We used the SSID to produce two cohorts. The first cohort (Grades 8-9) consists of 8th-grade students who took various CSTs mathematics, mainly the CST for General Mathematics and the CST for Algebra I in 2006. These cohort data were investigated for the differences between the 8th-grade CSTs for General Mathematics and Algebra I scores and students' performance on their following year's CST for Algebra I. The second cohort (Grades 7-8) consists of 7th-grade students who took the CST for Grade 7 Mathematics in 2006 and the CST for Algebra I at 8th grade in 2007. These cohort data were analyzed to determine predictive factors among 7th-grade CST mathematics sub-scores of the 8th-grade algebra achievement.

In the STAR program, most students take the sequence of CSTs for Algebra I, Geometry, Algebra II, and Summative High School Mathematics. In order to examine this college preparation pipeline, we analyzed students' participation in taking these CSTs between 2003 and 2011. We chose three cohorts of students, each with test-taking patterns for four year periods. Each cohort took the CST for Algebra at 8th grade, and, subsequently, they then took the CST for Geometry at 9th grade, Algebra II at 10th grade, and Summative High School Mathematics at 11th grade. These three cohorts provide the CST data for the analyses of this study from each of these years, 2003-2006, 2005-2008, and 2008-2011.

In the latest STAR report in 2012, 49% of 8th graders scored *proficient* or above on the CST for Algebra I (California Department of Education, 2012). The large failure rate of 8th graders on the CST and the studies that reveal misplacement of students in 8th-grade algebra courses (Kriegler & Lee 2006; Loveless, 2008; Taylor, 2011; Waterman, 2010; Williams et al., 2011) led us to investigate better pathways for algebra success. We took a close look at 8th-grade students and linked their 8th-grade test-taking and performance to their 9th-grade test-taking and performance with a longitudinal analysis. The majority of 8th-grade students constitute four subgroups: (1) those who took the CST for Algebra I and scored *proficient* or above; (2) those who took the

CST for Algebra I and scored below *proficient*; (3) those who took the CST for General Mathematics and scored *proficient* or above; and (4) those who took the CST for General Mathematics and scored below *proficient*.

Our focus is on the two marginal subgroups. We consider that group 1 is rightly placed in algebra and succeeds, and group 4 has little chance of succeeding in algebra because they failed a test that is much easier than algebra. We, then, name the groups in the middle on which we focus as (a) 8th-graders who scored below *proficient* on the CST for Algebra I and (b) 8th-graders who scored *proficient* or above on the CST for General Mathematics. We analyzed their 9th grade test-taking and performance.

In the quest to improve students' success in 8th-grade algebra, the overreaching question becomes what are the conditions for learning algebra and how do they relate to increasing students' success in learning algebra? One can address this question by examining the specific variables that predict students' algebra achievement. Our study focuses on such an investigation by examining variables of students' prior year's CST mathematics sub-scores. We were especially interested in finding out what prior year's scores on mathematics knowledge and skills might contribute to students' success in 8th-grade algebra. We identified specific sub-scores that reliably predict this success. Though outside the scope of our study, we acknowledge that some of the conditions for algebra success may include many other factors, besides those we examined, including the student's cognitive development (Gagné, 1963; Piaget & Garcia, 1989), their motivation (Middleton & Spanias, 1999), peer influence (Bulotsky-Shearer, Fernandez, Dominguez, & Rouse, 2011), school and community influence (Keck-Staley, 2010; Nasir, Hand, & Taylor, 2008; Cobb & Hodge, 2002), students' self-identity (Solomon, Lawson, & Croft, 2011; Nasir, 2002), and language proficiency (MacGregor & Price, 1999).

Historically, algebra has been a 9th-grade high school mathematics course (George, 2007), usually for students going to college and at other times for all students. It has also been viewed as a difficult subject to master (Heppel, 1895). Educators and policy makers have focused on academically preparing students to succeed in algebra, specifically and recently



through curriculum standards reform. In adopting the *California Mathematics Framework* in 2005, the State Board of Education approved a new list of standards, the Algebra Readiness Program, consisting of 16 California mathematics content standards. Among these standards, thirteen are grade 7 content standards and three are algebra I content standards (California Department of Education, 2006). The Algebra Readiness Program, recommended as a remedial course for students in grades 8 and 9, was designed to rebuild the foundational skills and concepts that might presumably be missing from students' academic learning in the early grades.

In March of 2008, the U.S. Department of Education released *Foundations for Success*, the final report by the National Mathematics Advisory Panel. The report lists the benchmarks for specific mathematics concepts and skills by grade level, which according to this report make up the "critical foundation of algebra" (U.S. Department of Education, 2008). The report calls on the nation's schools to provide "effective preparation of students for the study of algebra" (p. 15). According to the report, these concepts and skills are derived mainly from the following sources:

- 1) The Grades 1–8 curricula of the highest-performing countries on [Trends in International Mathematics and Science Study, added] TIMSS (Singapore, Japan, Korea, Hong Kong, Flemish Belgium, and the Czech Republic), sometimes called the "A+ countries,"
- 2) National Council of Teachers of Mathematics Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence (hereinafter Focal Points),
- 3) Grades K–8 in the six highest-rated state curriculum frameworks in mathematics,
- 4) a 2007 American College Testing (ACT) survey, and,
- 5) a Panel-sponsored survey of 743 teachers of introductory Algebra across the country who were asked what students need to learn to be prepared for success in Algebra. (p. 17)

Critics charged that the National Mathematics Advisory Panel's report lacked the empirical research

to back up its recommendations. As Thompson (2008) pointed out, those curricular content areas recommended by the panel were "based on the professional judgment of panel members, not on empirical research into teaching and learning algebra" (p. 582), and not on current research in learning and cognition, the science of learning. Therefore, studies are needed to explore the relationship between algebra experiences in courses, as seen in the necessary knowledge and skills to be learned, and algebra success. Specifically, it is of interest to examine the empirical links between students' prior content knowledge and skills and their learning success. In this study, we search for such a link, seeking answers to the question: What specific mathematics content knowledge and skills predict 8th-grade algebra achievement?

Again, we performed a longitudinal study with the data we earlier described, including a cohort of students when they were in 7th grade in 2006 and in 2007, when they were in 8th grade. As we have mentioned, we chose the data for these two years because 2006 is the last year when all 7th graders took the CST for Grade 7 Mathematics. This makes those CST scores for the year of Grade 7 Mathematics a complete data set. We matched the students in 8th grade who took the CST for Algebra I and found 208,043 matched records. We performed a linear multiple regression analysis using students' grade 7 CST sub-scores as independent variables and their CST for Algebra I raw scores as the dependent variable. There are 6 sub-scores (also called reporting clusters) in the CST for Grade 7 Mathematics:

- (1) *rational numbers*;
- (2) *exponents, powers, and roots*;
- (3) *quantitative relationship and evaluating expressions*;
- (4) *multistep problems, graphing, and function*;
- (5) *measurement and geometry*; and
- (6) *statistics, data analysis, and probability*.

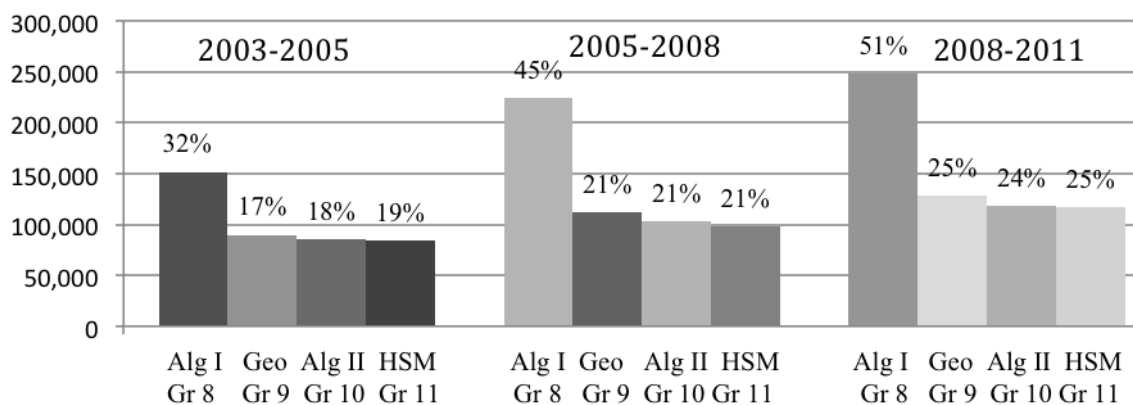
## Results

### *Question 1: The Impact of 8th-Grade Algebra on the College Preparation Pipeline*

Figure 1 shows numbers and percentages of students in grades 8 through 11 of three cohorts taking the California Standards Tests.

Two trends emerge from the chart: There are increases in the numbers of students taking algebra

Figure 1  
Numbers and Percentages of Students in Grades 8 Through 11 of Three Cohorts Taking the California Standards Tests.



and higher level CSTs across grade levels from 2003 to 2011, and the increases in the numbers of students in 9th through 11th grades taking higher level CSTs for mathematics are much smaller than the increases in the number of students in 8th grade taking the CST for Algebra I. For example, in 2003, 151,714 8th-grade students took the CST for Algebra I. By 2008, 248,155 8th-grade students took the CST for Algebra I. This increase involved an additional 96,441 students (about an additional 19 percentage points) of 8th-grade students taking the CST for Algebra I from 2003 to 2008. A more moderate pattern of increases exists for 9th- through 11th-grade students with regard to their participation in taking other higher level mathematics' CSTs between 2004 and 2011. For example, there are an additional 33,151 (about a 7 percentage points increase) 11th graders taking the CST for Summative High School Mathematics between 2006 and 2011. The existence of these two trends suggests that the desire of policy makers to increase attention and participation in algebra and higher-level mathematics appears to be having a desired effect.

Yet despite the impressive increases in 8th-grade students' taking the CST for Algebra I, there is not a corresponding increase in the numbers of students taking CSTs for higher mathematics. This fact suggests that these policies may be engendering increases in students' involvement in algebra, but not in the study of higher level mathematics in general. Students may not be able to or want to move beyond this entry-

level of higher mathematics' experience. While there have been increases in the numbers of students taking higher-level mathematics CSTs, they are not nearly as large as the increase in numbers of students taking the CST for Algebra I.

This deterioration between the number of 8th-graders CST for Algebra I takers and the number of 9th graders CST for Geometry takers signifies a decline and leads us to suggest that there may be a leak in the pipeline. It appears that simply encouraging more students to take 8th-grade algebra may not, by itself, lead to significantly more students taking advanced mathematics in high school.

The fact that the leaking pipeline of students' success in mathematics, beginning with 8th-grade algebra, shows deteriorations in the increase of higher level CSTs participation suggests that more has to be done than simply requiring a course or designating a set of knowledge and skills to be learned. Such encouragement for students to take courses is certainly necessary, but it is not sufficient for realizing students' understanding and encouraging their motivation to continue to learn higher mathematics.

The reductions in gains in students' participation in higher level CSTs for mathematics as well as the less than dynamic student performance on the CSTs students (not shown in the figure) through their high school grade level advancements led us to examine more closely student participation in 8th-grade mathematics classes, their passing or failing the CST,

Table 1

*Percentages of Students Scoring Proficient or Above in 9th-Grade Various CSTs Between Students who Scored below Proficient on the CST for Algebra I (subgroup A) and Students who Scored Proficient or Above on the CST for General Mathematics (subgroup B) at 8th Grade*

CST	Sub-group A %	Sub-group B %	Difference
General Mathematics	1.01	8.13	7.12
Algebra I	9.61	31.46	21.85
Geometry	2.72	1.27	-1.45
Total	13.67	41.01	27.34

and the correlations of these factors with higher scores on CST for Algebra I among 9th grade students.

*Question 2: Failing Algebra versus Passing General Mathematics at 8th Grade as Preparation for Higher Level Mathematics Study*

The majority of our two subgroups of students took the CST for Algebra I at 9th grade: 64% in subgroup A of the students scoring below *proficient* on the CST for Algebra I at 8th grade and 82% in subgroup B of the students scoring *proficient* or above on the CST for General Mathematics at 8th grade (not shown in the table). The second largest group of students in the rest of subgroups A and B took CST for General Mathematics with 8% and 12%, respectively; CST for Geometry with 27% and 4%, respectively. Table 1 shows the percentages of students scoring *proficient* or above in 9th grade on various CSTs between students in subgroup A and subgroup B.

As shown in Table 1, our subgroup B students outperform their peers in subgroup A significantly. Students who scored below *proficient* on the CST for Algebra I at 8th grade have much less chance of passing the CST for Algebra I at 9th grade compared to those students who scored *proficient* or above on the CST for General Mathematics (9.61% vs. 31.46%). In other words, those students who failed the CST for Algebra I at 8th grade and retook the same test at 9th grade had a 69% ( $1 - 0.0961/0.3146$ ) less chance of passing the test compared to those students who passed the CST for General Mathematics at 8th grade and took the CST for Algebra I at 9th grade for the first time. This striking

failure rate is highlighted in a California Department of Education press release that stated that for grades 8 through 11, only 15% of students repeating the CST for Algebra I scored *proficient* or above compared to 26% of first time algebra test-takers in all grades for the 2007 test administration. More recent data from the 2012 test administration show that 36% of first time Algebra I CST takers scored *proficient* or above compared to 24% of the re-takers scoring *proficient* or above (Torlakson, 2011, August 15, Table 6). The difference between first time algebra test-takers and repeaters in success rates and the fact that it appears to be continuing through 2012 raise serious questions about giving algebra one year sooner to those students, who scored below *proficient*. These rates also suggest that such a practice may not help them succeed in algebra in following years. If course placement can play a key role in providing students an appropriate education program and therefore lead to better success for their learning, we can then turn our attention to the conditions necessary for students' algebra success.

*Question 3: Linking Prior Year's Mathematics Knowledge and Skills to 8th-Grade Algebra Success*

Table 2 shows the multiple regression analysis of 8th-grade Algebra I's raw score using the CST for Grade 7 Mathematics' 6 reporting cluster sub-scores as predictors.

As indicated in Table 2, the sub-scores of 6 reporting clusters contributed 62% of the variance of the 8th graders' CST for Algebra I raw scores. If the CST for Grade

Table 2

*Multiple Regression Analysis of 8th-Grade CST for Algebra I's Raw Score Using the CST for Grade 7 Mathematics' 6 Reporting Cluster Sub-scores as Predictors (N=208,043)*

Variables	B	SE(B)	$\beta$	t	Sig. (p)
Rational numbers	.858	.008	.225	104.34	<.0001
Exponents, powers and roots	.963	.011	.165	86.53	<.0001
Quantitative relationship	.958	.010	.179	92.48	<.0001
Multistep problems and graphing	.502	.009	.128	58.57	<.0001
Measurement and geometry	.730	.008	.184	87.22	<.0001
Statistics and analysis	.640	.015	.075	42.69	<.0001

$R^2 = .620$

7 Mathematics sub-scores predicts 8th-grade algebra achievement with great reliability, then, what subsets of skills and knowledge might contribute to this strong prediction? Our model indicates that the sub-score of the reporting cluster *rational numbers* is the strongest predictor, contributing 48% (not shown in the table) of the variance of the 8th-grade CST for Algebra I scores, with a Beta ( $\beta$ ) value of .225. That is, a one-unit *standard deviation (SD)* increase in the CST for the sub-score of *rational numbers* results in .225 SD units' increase of the CST for Algebra I. The second strongest predictor is the sub-score of *quantitative relationships and evaluating expression*, contributing 8% (not shown in the table) of the variance of 8th-graders' CST for Algebra I scores, with a Beta value of .179.

According to the California Department of Education (California Department of Education, 2009), the *rational numbers* reporting cluster of the CST for Grade 7 Mathematics assesses whether students "know the properties of, and compute with, rational numbers expressed in a variety of forms" (p. 2). The various forms of these rational numbers include integers, fractions, decimals, and percents (California Department of Education, 2006). In the released test questions 1-20 (California Department of Education, 2009, pp. 7-12), one can observe that the subset of *rational numbers* tests students' ability to manipulate fractions, decimals, and percents.

One statistical concern is that all the sub-scores in the six reporting clusters are highly correlated. This

multicollinear relationship of the sub-scores could be seen as inflating the variance and distorting the relationship between predictor variables and criterion. However, this multicollinearity factor is monitored by the variance inflation factors (VIF), which range from 1.7 to 2.6 (not shown in Table 2) for the sub-scores of the six reporting clusters in the prediction model. Being smaller than ten, these VIF of the sub-scores do not affect the predicted values because only VIF numbers larger than ten are considered large enough to affect the predicted values (SAS, 2004).

### Discussion

The CSTs results from our analysis show that the increase in the numbers of 8th graders taking algebra has indeed expanded the college preparation pipeline in high schools. However, this pipeline has a significant leak in it. In the efforts to focus on improving students' learning in middle schools and high schools, we must look at each and every one of the students we serve and the empirical and theoretical evidence to determine the most effective learning conditions for enhancing student learning.

Our study, and many others, has shown that placing all 8th-grade students into algebra courses does not help all of them in their subsequent year's of learning (Clotfelter, Ladd, & Vigdor, 2012; Liang, Heckman, & Abedi, 2012; Loveless, 2008; Taylor, 2011; Waterman, 2010; Williams, Haertel, Kirst, Rosin, & Perry 2011; Williams, et al., 2011). Our study also indicates

that students who scored below *proficient* on the CST for Algebra I at 8th grade have a 69% less chance to score *proficient* or above on their CST for Algebra I at 9th grade compared to their peers who scored *proficient* or above on the CST for General Mathematics at 8th grade. Our regression model reveals that, among the content subject area, *rational numbers* is a strong predictor of algebra achievement.

This finding about the importance of rational number sense fits with other studies of learning conditions that support student understanding of mathematics. Scardamalia et al. (2012) concluded that, “proportional thinking or rational number sense is more fundamental and more skill-enhancing than mastering (or not quite mastering) a number of rational number algorithms” (p. 233). The reason for this admonition and consideration has been known and argued for in the research literature for some time. Davis (1994) noted the following:

What seems to be the newly emerging view is that the goal of teaching mathematics deals primarily with how students think about various kinds of problems, and with providing students with enough meaningful (and often concrete) experience so that students can build up, in their own minds, a large and powerful repertoire of basic metaphors or precursors of mathematical ideas (or, if you prefer, assimilation paradigms). These are the mental tools that make it possible for students to build mental representations or problem situations, and representations of possibly helpful knowledge. In short, these are the building blocks with which a student can think mathematically. (p. 613)

It is important to point out that our study is only based on the data of test results. We do not know students’ educational programs, nor do we know their efforts and their teachers’ efforts in classrooms, other than their demographic characteristics. The algebra curriculum that is being taught in the schools across California and the nation today is, by and large, a curriculum of classical algebra that has changed little from the one taught in American high schools in the late

1800s. The difference today from that of the late 19th century is that all of the students are being asked to succeed on tests in algebra. In the California STAR data, there are many students repeating the CST for Algebra I, once, twice, three times, and in extreme cases, four times. It is highly possible that some of these repeaters failed the courses within the same educational settings and with the same curricula that have been repeated many times. In order to improve the much weaker chance of success in the second time trying the CST for Algebra I, as indicated in our study, educators and policy makers need to turn their attention to a broader scope, such as educational settings, curricula, and pedagogy and allow alternatives to the ones our students have experienced without much success. One of the alternatives to these persistent practices is in the creation of educational settings and conditions for Indigenous Invention (Heckman & Montera, 2009), which encourages educators and students with whom they work to be creators, inventors, and innovators. The alternative curricula that arise in such an endeavor unleash teachers’ creativity and students’ funds of knowledge (Gonzales, Moll, & Amanti, 2005).

A recent educational movement in the nation, as well as in California, is the adoption of the Common Core State Standards (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The new standards list algebra as part of high school standards. Standards for grade 8 mainly focus on arithmetic and pre-algebra. In 2010, when the California Academic Content Standards Commission adopted the common core standards, it added California 8th grade Algebra I standards (California Department of Education, 2010). The commission recognized that not all California 8th graders would be successful in algebra and prescribed dual standards for 8th grade. However, a new bill (SB 1200), signed by Governor Brown in September, 2012, allows the state to approve or modify the common core academic content standards in mathematics (California Education Code, Section 60605.11(a). The law also specifies “One set of standards is adopted at each grade level” (California Education Code, Section 60605.11(b)(3). While critics charge this is a backward move, the President of State Board of Education Michael Kirst stated that SB 1200 “marks a critical step forward in California’s efforts to



implement the Common Core Standards and to ensure algebra is accessible to every student" (Fensterwald, 2012, September 28). Dr. Kirst also clarified this point in an Education Week article (Robelen, 2012, October 24), as saying that the new law, "allows the state to clarify that it will provide two course pathways, one for students ready for algebra at grade 8 and another for those who take it a year later" (p. 11).

In January 2013, the State Board of Education adopted a revision of California Common Core Standards. This revision stripped California's Algebra I standards from the state's 8th- grade math standards (Fensterwald, 2013, January 17). In the current development of California mathematics framework, the state will create curriculum options to accelerate math-taking in middle school and high school and to leave it up to local districts to determine who's eligible for them. One month later, the advisory committee to the Public School Accountability Act (PSAA) recommended dropping the API penalties for schools and districts that do not test 8th grade students in algebra (Fensterwald, 2013, February 13). In March 2013, the State Board of Education approved the elimination of the penalty of testing 8th and 9th graders in general mathematics for schools and districts accountability (State Board of Education, 2013). By doing so, it ended the California 8th grade algebra for all policy.

Our findings suggest that questioning the basic frameworks that have guided the development of learning in school mathematics and the policies for advancing student achievement in mathematics would make greater contributions to student learning success than creating requirements that are based on unwarranted claims. Without empirical evidence, what works and what does not work in educational reform appears to depend then on arguments based in beliefs and unwarranted claims. Instead, we recommend further investigations that will permit a better understanding of the limits and the range of opportunities for our diverse students in California and their success in this very important and critical area of mathematics. It is not enough to use carrots and sticks in making students trudge through a presumed list of important knowledge and skills and learning activities that do not yield the promises given to them. We are better than that as a State, as educators, researchers,

and citizens.

#### **Author Note**

The opinions expressed by Jian-Hua Liang are of the author alone and do not reflect opinion or policy of the California Department of Education.

#### **About the Authors**

*JIAN-HUA LIANG is an education research and evaluation consultant with the California Department of Education. Her research interests include evaluation, curriculum, and assessment with an emphasis on mathematics.*

*email: [jliang@cde.ca.gov](mailto:jliang@cde.ca.gov)*

*PAUL HECKMAN is a Professor and Associate Dean in the School of Education at the University of California, Davis. His research interests focus on educational change and invention in urban communities, schools, and after school programs. His articles and book chapters address issues of change and invention in curriculum and organizational structures urban settings.*

*email: [peheckman@ucdavis.edu](mailto:peheckman@ucdavis.edu)*

## REFERENCES

- American Law Institute (1988-2009). *Restatement (Third) of the foreign relations law of the United States*. Philadelphia, PA: American Law Institute. Eagan, MN: West-Thompson.
- Aiken, W.M. (1942). *The story of the eight-year study: With conclusion and recommendations*. New York: Harper & Brothers.
- Allensworth, E.M. & Nomi, T. (2009). *College-preparatory curriculum for all: The consequences of raising mathematics graduation requirements on students' course taking and outcomes in Chicago*. Paper presented at the Second Annual Conference of the Society for Research on Education Effectiveness, March 2009, Arlington, VA. Retrieved September 20, 2009, from [http://www.sree.org/conferences/2009/pages/abstracts/055\\_college.doc](http://www.sree.org/conferences/2009/pages/abstracts/055_college.doc)
- Bozick, R., & Owings, J. A. (2008). *Mathematics coursetaking and achievement at the end of high school: Evidence from the education longitudinal study of 2002 (ELS: 2002)*. Washington DC: National Center for Education Statistics (U.S. Department of Education).
- Bradley, A. (1994, May 11). Cortines raises graduation requirements in N.Y.C. *Education Week*.
- Bulotsky-Shearer, R.J., Fernandez, V., Dominguez, X., & Rouse, H.L. (2011). Behavior problems in learning activities and social interactions in head start classrooms and early reading, mathematics, and approaches to learning. *School Psychology Review, 40(1)*, 39-56.
- California Department of Education. (2003). *Standardized Testing and Reporting (STAR) 2003 test results*. Retrieved December 1, 2012, from <http://star.cde.ca.gov/star2003>
- California Department of Education, (2006). *Mathematics frameworks for California public schools: Kindergarten through grades twelve*. Retrieved August 4, 2008, from <http://www.cde.ca.gov/ci/ma/cf/documents/mathfrwkcomplete.pdf>
- California Department of Education. (2007). *Standardized Testing and Reporting (STAR) 2007 test results*. Retrieved December 1, 2012, from <http://star.cde.ca.gov/star2007>
- California Department of Education. (2008). *2007-2008 Academic performance index: Information guide*. Retrieved February 21, 2009, from <http://www.cde.ca.gov/ta/ac/ap/documents/infoguide07b.pdf>
- California Department of Education. (2009). *Released Test Questions for the CST for Grade Seven Mathematics*. Retrieved June 29, 2012, from <http://www.cde.ca.gov/ta/tg/sr/documents/cstrtqmath7.pdf>
- California Department of Education. (2010). *California common core state standards*. Retrieved December 2, 2012, from [http://www.scoe.net/castandards/agenda/2010/math\\_ccs\\_recommendations.pdf](http://www.scoe.net/castandards/agenda/2010/math_ccs_recommendations.pdf)
- California Department of Education. (2011). *Standardized Testing and Reporting (STAR) 2011 test results*. Retrieved December 1, 2012, from <http://star.cde.ca.gov/star2011>
- California Department of Education. (2012). *Standardized Testing and Reporting (STAR) 2012 test results*. Retrieved December 1, 2012, from <http://star.cde.ca.gov/star2012>
- Center for the Future of Teaching & Learning. (2005). *California's approach to math instruction doesn't add up*. Santa Cruz, CA. (ERIC Document Reproduction Service No.: ED485345)
- Chamberlin, D., Chamberlin, E.S., Drought, N.E., & Scott, W.E. (1942). *Did they succeed? The follow-up study of the graduates of the thirty schools*. New York: Harper & Brothers.
- Charters, W.W., & Jones, J.E. (1973). On the risk of appraising non-events in program evaluation. *Educational Researcher, 2(11)*, 5-7.
- Clotfelter, C.T., Ladd, H.F., & Vigdor, J.L. (2012). *The aftermath of accelerating algebra: Evidence from a district policy initiative*. (CALDER Working Paper No. 69). Washington, DC: American Institute of Research.
- Cobb, P. & Hodge, L.L. (2002). A relational perspective on issues of cultural diversity and equity as they play out in the mathematics classroom. *Mathematical Thinking and Learning, 4(2&3)*, 249-284.
- College Board. (2000). *Equity 2000: A systemic education reform model, a summary report, 1990-2000*. Retrieved July 31, 2008, from [http://www.collegeboard.com/prod\\_downloads/about/association/equity/EquityHistoricalReport.pdf](http://www.collegeboard.com/prod_downloads/about/association/equity/EquityHistoricalReport.pdf)
- Cooney, T.J. (1988). The issue of reform: What have we learned from yesteryear? *The Mathematics Teacher, 81(5)*, 352-363.
- Cremin, L. (1955). The revolution in American secondary education, 1893-1918. *Teachers College Record, 56(4)*, 295-308.
- Cremin, L. (1964). *The transformation of the school: Progressivism in American education*. New York: Vintage Books.
- Davis, R. B. (1994). The task of improving mathematics classrooms: A reply to Schofield, Eurich-Fulcer, and Britt. *American Educational Research Journal, 31(3)*, 608-618.
- Fensterwald, J. (2012, September 28). *State board gets authority to pare back eighth grade math standards*. Oakland, CA: EdSource. Retrieved December 2, 2012, from <http://www.edsource.org/today/2012/state-board-gets-authority-to-pare-back-8th-grade-math-standards/20533>
- Fensterwald, J. (2013, January 17). *It's final: State board shifts policy on eighth grade algebra*. Oakland, CA: EdSource. Retrieved February 18, 2013 from <http://www.edsource.org/today/2013/its-final-state-board-shifts-policy-on-eighth-grade-algebra/25672>
- Fensterwald, J. (2013, February 13). *API penalties for not offering 8th grade algebra to be dropped*. Oakland, CA: EdSource. Retrieved February 18, 2013, from <http://www.edsource.org/today/2013/api-penalties-for-not-offering-8th-grade-algebra-to-be-dropped/27063>
- Gagné, R. M. (1963). Learning and proficiency in mathematics. *The Mathematics Teacher, 56*, 620-626.
- Gamoran, A., & Hannigan, E.C. (2000). Algebra for everyone? Benefits of college-preparatory mathematics for students with diverse abilities in early secondary school. *Educational Evaluation and Policy Analysis, 22(3)*, 241-254.
- George, M. (2007). The history of liberal arts mathematics. *Dissertation Abstracts International, 68(11)*, 192. (UMI No. 3288599)

- Goodlad, J.I., Klein, F., & Associates (1970). *Looking behind the classroom door*. Worthington, Ohio: C.A. Jones Publishing Co.
- Gonzalez, N., Moll, L.C., & Amanti, C. (Eds.) (2005). *Funds of knowledge: Theorizing, practices in households, communities and classrooms*. Mahwah, New Jersey: Lawrence Erlbaum Associates, Inc.
- Heckman, P.E., & Montera, V.L. (2009). School reform: The flatworm in a flat world: From entropy to renewal through indigenous invention. *Teachers College Record*, 111(5), 1328–1351.
- Heppel, G. (1895). Algebra in schools. *The Mathematical Gazette*, 4, 25-27.
- Herrera, T.A., & Owens, D.T. (2001). The “New Math”? Two reform movements in mathematics education. *Theory into Practice*, 40(2), 84-92.
- Hiebert, J. (Jan, 1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education*, 30(1), 3-19.
- Keck-Staley, T.L. (2010). The role of human resource capital of Black and Latino middle schoolers’ mathematics identities. *The Negro Educational Review*, 60(1-4), 7-40.
- Kriegler, S., & Lee, T. (2006). *Using standardized test data as guidance for placement into 8th grade algebra*. Los Angeles, CA: UCLA Math Content for Teachers. Retrieved October 11, 2008 from: [http://www.introtoalg.org/downloads/Algebra\\_8th\\_Grade\\_Paper.pdf](http://www.introtoalg.org/downloads/Algebra_8th_Grade_Paper.pdf)
- Liang, J.-H., & Guo, S. (2007). *A preliminary study of the California mathematics standards test-taking patterns*. Paper presented at the 86th Annual California Educational Research Association Conference, Dana Point, CA.
- Liang, J.-H. (2009). Linking eighth- and ninth-grade algebra success to key variables of prior mathematics knowledge and skills: A predictive and comparative analysis. *Dissertation Abstracts International*, 70(08), 159. (UMI No. 3369854)
- Liang, J.-H., Heckman, P.E., & Abedi, J. (2012). What do the California standards test results reveal about the movement toward eighth-grade algebra for all? *Educational Evaluation and Policy Analysis*, 34(3), 328-343.
- Loveless, T. (2008). *The misplaced math student: Lost in eighth-grade algebra*. Washington, DC: Brookings Institute.
- MacGregor, M., & Price, E. (1999). An exploration of aspects of language proficiency and algebra Learning, *Journal for Research in Mathematics Education*, 30(4), 449-467.
- McLaughlin, D. H., Cohen, J., & Lee, R. (1997). *NELS: 88 survey item evaluation report*. Washington DC: U.S. Department of Education.
- Middleton, J.A., & Spanias, P.A. (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal of Research in Mathematics Education*, 30(1), 65-88.
- Moses, R.P., Kamii, M., Swap, S.M., & Howard, J. (1989). The algebra project: Organizing in the spirit of Ella. *Harvard Educational Review*, 59(4), 423-443.
- National Center for Education Statistics. (2005). *NEAP 2004 trends in academic progress: Three decades of student performance in reading and mathematics: Findings in brief*. (NCES 2005-463). Washington, DC: U.S. Department of Education.
- The National Commission on Excellence in Education. (1983). *A nation at risk: The imperative for educational reform*. Washington, DC: U.S. Government Printing Office
- National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core state standards for mathematics*. Washington, DC: Author. Retrieved May 8, 2012, from <http://www.corestandards.org/the-standards>.
- Nasir, N.S. (2002). Identity, goals, and learning: Mathematics in cultural practice. *Mathematical Thinking and learning*, 4(2&3), 213-247.
- Nasir, N.S., Hand, V., & Taylor, E.V. (2008). Culture and mathematics in school: Boundaries between “cultural” and “domain” knowledge in the mathematics classroom and beyond. *Review of Research in Education*, 32, 187-240.
- Piaget, J. & Garcia, R. (1989). *Psychogenesis and the history of science*. (Helga Feider, Trans.). New York: Columbia University Press.
- Robelen, E. (2012, October 24). California law shift gears on algebra, textbooks. *Education Week*, 32(9), pp. 1 and 11.
- Oakes, J. (1985). *Keeping track: How schools structure inequality*. New Haven, CT: Yale University Press.
- Oakes, J. (1990). *Multiplying inequalities: The effects of race, social class, and tracking on opportunities to learn mathematics and science*. Santa Monica, CA: RAND Corporation.
- Oakes, J., & Guiton, G. (1995). Matching: The dynamics of high school tracking decisions. *American Education Research Journal*, 32(1), 3-33.
- Ravitch, D. (2000). *Left Back*. New York: Simon and Schuster.
- Riley, R.W. (1997). *Mathematics equals opportunity*. White paper prepared by the U.S. Secretary of Education Richard Riley. Washington, DC: U.S. Department of Education.
- Romberg, R.A. (1992). The scholarly basis of the school mathematics reform movement in the United States. *International Journal of Educational Research*, 17(5), 419-437.
- SAS Institute Inc. (2004). *SAS/STAT 9.1 user's guide*. Cary, NC: author.
- Scardamalia, M., Bransford, J.S., Kozma, B., & Quellmatz, E. (2012). New assessments and environments for knowledge building. In Griffin, P., McGaw, B., & Care, E. (Eds.), *Assessment and teaching of 21st century skills* (pp. 231-300). New York: Springer Dordrecht Heidelberg.
- Shulock, N., & Moore, C. (2007). *Rules of the game: How state policy creates barriers to degree completion and impedes student success in the California community college?* Sacramento, CA: Institute for Higher Education Leadership & Policy.
- Siegler, R.S., Duncan, G.J., Davis-Kean, P.E., Duckworth, K., Claessens, A., Engel, Susperreguy, M.I., & Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science*. Retrieved June 14, 2012. DOI: 10.1177/0956797612440101.
- Silva, C.M., Moses, R.P., Rivers, J., & Johnson, P. (Summer, 1990). The Algebra project: Making middle school mathematics count. *The Journal of Negro Education*, 59(3), 375-391.

- Smith, J. (1996). Does an extra year make any difference? The impact of early access to algebra on long-term gains in mathematics achievement. *Educational Evaluation and Policy Analysis, 18*(2), 141-153.
- Solomon, Y., Lawson, D., & Croft, T. (2011). Dealing with "fragile identity": Resistance and refiguring in women mathematics students. *Gender & Education, 23*(5), 565-583.
- Spencer, H. (1860). *What knowledge is of most worth? In education-intellectual, moral and physical*. New York: D. Appleton and Co.
- Spielhagen, F.R. (2006). Closing the achievement gap in math: The long-term effects of eighth-grade algebra. *Journal of Advanced Academics, 18*, 34-59.
- Stanic, G., & Kilpatrick, J. (1992). Mathematics curriculum reform in the United States: A historic perspective. *International Journal of Educational Research, 17*(5), 407-417.
- State Board of Education, (2013). *SBE meeting for March 2013*. CA: California Department of Education. Retrieved March 13, 2013, from <http://www.cde.ca.gov/be/ag/ag/main201303.asp>
- Stevenson, D. L., Schille, K. S., & Schneider, B. (1994). Sequences of opportunities for learning. *Sociology of Education, 67*(3), 184-198.
- Taylor, D.J. (2011). Outcomes of placing low performing eighth grade students in algebra content courses. *Dissertation Abstracts International, 73*(01), 125. (UMI No. 3474480)
- Thompson, P. W. (2008). On professional judgment and the national mathematics advisory panel report: Curricular content. *Educational Researcher, 37*(9), 582-587.
- Torlakson, T. (2012, August 15). *2012 STAR results mark ninth straight year of rising English and math scores*. (News Release: #12-79). Sacramento, CA: California Department of Education. Retrieved November 30, 2012, from <http://www.cde.ca.gov/nr/ne/yr12/yr12rel79.asp>
- Tyack D., & Cuban, L. (1995). *Tinkering toward utopia: A century of public school reform*. Cambridge, MA: Harvard University Press.
- U. S. Department of Education. (2008). *Foundations for success: Report of the national mathematics advisory panel*. Retrieved January. 25, 2009, from <http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf>
- University of California. (2007). *Subject requirement ("A-G" coursework)*. Updated May 31, 2007. Retrieved July 28, 2008, from [http://www.universityofcalifornia.edu/admissions/undergrad\\_adm/paths\\_to\\_adm/freshman/subject\\_reqs.html](http://www.universityofcalifornia.edu/admissions/undergrad_adm/paths_to_adm/freshman/subject_reqs.html)
- Viadero, D. (2009, March 11). Algebra-for-all policy found to raise rates of failure in Chicago. *Education Week*.
- Waterman, S. (2010). *Pathways report: Dead ends and wrong turns on the path through Algebra*. Palo Alto, CA: Noyce Foundation. Retrieved on January 3, 2012, from [http://www.noycefdn.org/documents/Pathways\\_Report.pdf](http://www.noycefdn.org/documents/Pathways_Report.pdf)
- Williams, T., Haertel, E., Kirst, M.W., Rosin, M., & Perry, M. (2011). *Preparation, placement, proficiency: Improving middle grades math performance. Policy and Practice Brief*. Mountain View, CA: EdSource.

