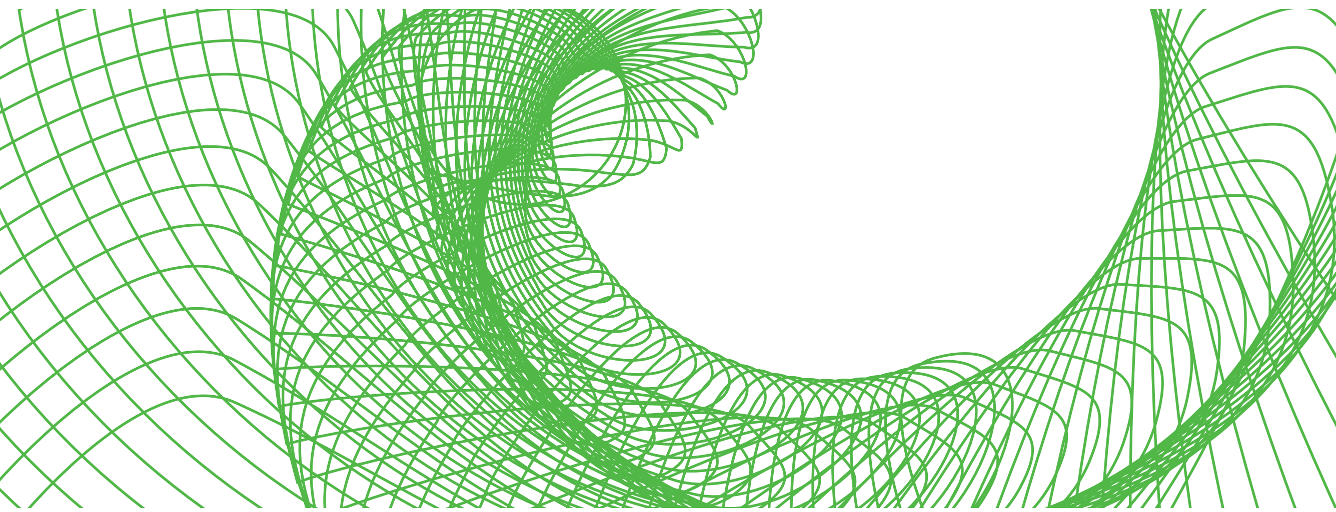
journal of the central california mathematics project

VOLUME 2



California State University | Stanislaus

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Foreword to Volume 2

It is with great joy that we present to you the second volume of the Journal of the Central California Mathematics Project. This publication is made possible with financial support from the California mathematics Project, Stanislaus County Office of Education and California State University, Stanislaus.

The second volume shares the basic characteristics of the first volume in the nature and content of the article. Kerri Khan, CSU Stanislaus student, with assistance from Math instructor Natalia Moore and Professor Viji Sundar presents an activity to help students become discriminating consumers. CSU Stanislaus Instructor Maryam Arvizu talks about her experience on the Supplemental Instruction. Professor Angelo Segalla from CSU Long Beach and Professor Shandy Hauk from University of Northern Colorado introduce WeBWorK; this is the first of three installments on WeBWorK in future CCMP Journals. Merced College Instructor Stephanie Souza and High School teacher Renukha Prakash discuss some of the common errors made by algebra students and how to prevent them. Professor Jung-Ha An introduces students to mathematical modeling using Difference Equations. Francisco Garcia from Los Banos High offers teachers Twelve Challenging Problems to review and reteach in algebra and geometry classes. Stella Estrada and Viji Sundar present a strategy to make the multiplication facts music for your ears.

I am grateful to the reviewers who were generous with their time and expertise.

Viji K. Sundar, Professor of Mathematics
Director, Mathematics Grants and Sponsored Programs
California State University, Stanislaus

Introduction from Volume 1

The mission of the California Mathematics Project (CMP) is to enhance K-12 teachers' content knowledge and instructional strategies aligned with the *California Mathematics Content Standards* and *Framework* through the collaboration between mathematicians, mathematics educators, and teacher leaders.

The Central California Mathematics Project (CCMP) at CSU Stanislaus, one of 19 CMP sites, has been a CMP site since 1984. Throughout the 24 years, CCMP has served many teachers and students, providing a variety of professional development programs to increase teachers' effectiveness in the classroom. CCMP has provided programs for schools and districts including teachers of English learners, teachers seeking subject matter competency, and teachers in high need schools and districts. During many of their summer programs, teachers from Thailand were invited to participate. This partnership with Thailand has broadened the perspectives of all teachers participating.

This inaugural CCMP journal is the result of years of effort, discovering what works for teachers. CCMP is sharing what has worked for them. To prepare for the writing of this journal, CCMP held a summer institute during 2008 for participants interested in writing articles for this journal. Consultants with expertise in mathematics and/or writing supported the participants, reading many drafts and providing technical assistance. I had the privilege of visiting CCMP during this institute and fit right in since I had a writing task to complete that week. It was great to see the participants so engaged in their writing efforts, so much that they were often oblivious to their surroundings.

From elementary classroom activities to overviews of university coursework, this journal has something for all mathematics educators and teachers of all grade levels. It is a credit to Dr. Viji Sundar, Director and Faculty Advisory of CCMP, for her vision to accomplish this work.

Enjoy reading and using the activities.

Susie W. Hakansson, Ph.D.
Executive Director
California Mathematics Project

Foreword from Volume 1

One of the primary goals of the Central California Mathematics Project is to "Develop instructional strategies to improve academic performance of students in mathematics." This collection contributes a great deal to this goal, while it also reinforces and exemplifies the commitment to excellence in teaching at California State University, Stanislaus and throughout the six-county region that the CCMP serves.

As an internal publication of the CCMP, this collection features math lessons and best practices that have been classroom tested, and which will surely empower both teachers and students. With the assistance of Professor Viji Sundar, CSU Stanislaus student Deidre Rodriguez provides an engaging approach to drawing the dimensions of sports fields; Loma Linda Junior High School math teacher Mary Nay gets at fundamental math concepts through popular comic strips; Ceres High School math teacher Ramina Isaac helps students see that math is a key to understanding real-world problems; Caswell Elementary School teacher Mary Gonzales provides an amusing and memorable poem that helps third-graders learn to round numbers; and Merced College math instructor Stephanie Souza points us toward career skills, with an intensive math course developed for Forest Service employees. In addition, CSU Stanislaus Professor Heather Coughlin presents both an essay on the challenges elementary school teachers face when creating word problems that involve dividing fractions, and an engaging class activity that involves students in calorie-counting and price comparisons. Further representing the breadth of interest and participation in this project, Professor Ruangporn Prasitkusol from Phranakhon Rajabhat University offers a model university course in "Statistics for Research; Dr. Wimol Sanguanwong, from Wat Pra Srimahadhat Secondary Demonstration School in Bangkok presents approaches to teaching quartiles to high school students, and Viji Sundar and Marie Vanisko (from Carroll College in Helena, Montana) present a course in linear and abstract algebra for non-math majors.

The range of topics in this collection is impressive, and is designed to appeal to a number of teachers in our region. In addition, it exemplifies collaborative energy, provides a valuable look at effective approaches to pedagogy and curricular development, and re-emphasizes the dedication to student success that we all cherish. I am very grateful to Professor Sundar and her colleagues for putting together this collection, and trust that you will all read and profit from its lessons and advice.

William A. Covino
Provost and Vice President for Academic Affairs
California State University, Stanislaus

Comparing Apples to Oranges

Kerri Khan, California State University, Stanislaus

Natalia Moore, California State University, Stanislaus

Viji Sundar, California State University, Stanislaus

CONCEPTS: Number Sense, Measurement, Algebraic Thinking, Mathematical Reasoning

SKILLS: Comparing, weighing, data collecting, counting, writing, calculating, unit pricing, critical thinking.

GRADE LEVEL: 6 - 7

MATERIALS: kitchen scale, plastic knives, paper towels, plastic bowls, plastic Ziploc™ bags, fresh fruit (apples, bananas, grapes, oranges), Activity Sheet, pencils, calculators.

BACKGROUND: Are bananas less expensive than grapes? Are oranges more expensive than apples? Which fruit gives the consumer the best value? The goal of this project is to help students become discriminating consumers. This activity is an applied mathematics lesson about comparing the cost of different fruit purchased at the market. By the end of the activity students will know how to find the cost of the edible part of each type of fruit so that they may compare and determine which gives the best value by weight. In this project, students will get the opportunity to learn that math is more than just numbers and surrounds our every day experiences. Studies show that students find learning mathematics to be interesting and fun when applied to real life situations. Hands-on math problems that engage students help them visualize the concept and give a deeper understanding of the concepts than listening to a lecture or seeing a demonstration.

DESCRIPTION: The project begins with the following teacher-led discussion.

<u>The first step</u> is a class discussion to distinguish the edible from the non-edible part of each fruit. By the end of this step the class should be in agreement that each purchased fruit is made up of two parts: an edible portion and a non-edible portion. Students should also be clear as to which portion of each of the fruits purchased is defined as edible for this project. Students may offer their own thoughts of what they think will be the edible portion of each fruit. The teacher should record student responses on the board or a chart so that they can be compared with the actual results once the project is completed.

<u>The second step</u> is to establish a consistent way of measuring the cost of the edible portion of each fruit so that we may compare the costs. For the purpose of this paper we will only consider the edible part of the fruit to be of 'value'. The cost of different fruit can then be compared to the of the cost per pound based on the price paid and the weight of the edible portion of the fruit. To find the unit price or "real cost per pound" of each, we will divide the price paid for the fruit by the weight of the edible portion of the fruit. Students may add to their previous predictions, which fruit they think will have the lowest "real cost per pound".

<u>The third step</u> is to divide the class into four groups - one group for each fruit. We will use four common fruits – bananas, oranges, apples and grapes. Each group may be assigned a fruit to bring in for the project. Students must also make a note of the market price per pound for each type of fruit. Each group will be responsible for the weight of the purchased fruit as is at the start of the project. Once this weight is recorded on the Activity Sheet, each group may then separate the edible from the non-edible portion – clean, peel, de-seed the fruit. After completing this step, students weigh and record the weight of the edible portion on the activity sheet. Once this data is recorded on the Activity Sheet, students may perform the calculations necessary to determine the "real cost per pound" of the fruit. Below is a description of the edible and non-edible part of these fruits.

Banana: The banana has a protective outer layer that is known as the peel and the fleshy edible inner portion. For this project the edible part will be the fleshy inner portion and non-edible will be the peel.

Orange: The orange has a thick outer peel called the rind and the inner, edible portion that is divided into segments. For this project the edible part will be the inner portion and the non-edible will be the peel which will be discarded

Apple: The apple has a thin outside layer called the skin. On the inside of the apple there is the edible fleshy portion, and the non-edible core, and seeds. For this project the non-edible portion will be the core and the seeds which will be discarded.

Grapes: Grapes usually come in a cluster attached to the stem. For this project the edible part will be the entire grape and the non-edible portion will be the stem.

Charting Information

Hand out Activity Sheet to students. Have students chart the information.

- Write the name of the fruit (column 1) and the market price per pound, which is the unit price (column 2).
- Weigh all pieces of specified fruit (non-peeled with skin) and chart weight (column 3).

- Use the plastic knife to peel, de-seed each fruit.
- Weigh ready to eat fruit (edible portion) then record weight (column 5).

Once the above data is collected, the calculations may be completed.

- Find the cost of the fruit purchased by multiplying the price per pound (column 2) by the numbers of pounds (column 3) and record the result (column 4).
- To find the "real cost per pound", divide the cost of the fruit (column 4) by the number of pounds of edible fruit (column 5) and record the result (column 6).

After each group has completed the above steps, compare the "real cost per pound" of each of the fruits' edible portion (column 6).

- Decide which gives the best price of all the fruits purchased.
- Check back to predictions and compare to the actual results of the project.

Extension: The above experiment can be repeated for the other consumer products. For example, students can compare the prices of:

- fresh whole vegetables with ready to eat cut up vegetables like carrots, broccoli, spinach etc.
- fresh vegetables with ready to cook frozen vegetables like carrots, broccoli, spinach etc.
- fresh vegetables with canned vegetables

Activity Sheet

Purchased Price				Edible Portion Cost	
(1) Name of Fruit	(2) Market Price per Pound	(3) Weight	(4) Market Cost $(2) \times (3) = (4)$	(5) Weight	(6) My Real Cost $(4) \div (5) = (6)$
PEAR	$\$1.50/lb$	$0.5\,lb$	$\$1.50/lb \times 0.5\,lb = \$0.75$	$0.25\,lb$	$\frac{\$0.75}{0.25\,lb} = \frac{\$3.00}{lb}$

The Supplemental Instruction Program and My Experience

Maryam Arvizu, California State University, Stanislaus

I am always looking for ways to improve my teaching and part of this approach is to try different resources.

I became aware of Supplemental Instruction program a while ago and believe that it is one of the greatest resources for helping students to succeed. It provides guided study sessions where students learn new study strategies in the context of course concepts.

I would like to start by offering an overview of the SI program.

SI Program Overview

1) Definition:

> Supplemental Instruction (SI) is an academic assistance program that utilizes peer-assisted study sessions. SI sessions are regularly scheduled, informal reviews where students compare notes, discuss readings, develop organizational tools, and predict test items. Students learn how to integrate course content and improve their study skills while working together. The sessions are facilitated by "SI Leaders," students who have previously done well in the course and who attend all class lectures, take notes, and act as model students.

2) Purpose:

- to increase retention within targeted historically difficult courses
- to improve student grades in targeted historically difficult courses
- to increase the graduation rates of students

3) Participants:

> SI is a "free service" offered to all students in a targeted course. Students with varying levels of academic preparedness are encouraged to attend this voluntary program. Participants come from a diverse range of ethnic backgrounds. SI is a not a developmental approach to learning as the program targets high-risk, courses rather than high-risk students, and because of that there is no remedial stigma attached.

How SI Works

The SI model involves four key roles – SI Coordinator, SI Faculty, SI Student Leaders, and Participating Students.

1. *The SI Coordinator* is a trained professional who is responsible for identifying the targeted courses, cultivating faculty support, selecting and training SI leaders, as well as marketing and evaluating the program on an ongoing basis.
2. *The faculty members* of the identified historically difficult courses invite and support SI. Faculty members screen SI leaders for content competency and approve leader selections, and collaborate with the SI leaders and Coordinator on a regular basis.
3. *The SI leaders (near peers)* are students who have demonstrated core course competency and have been approved by the course instructor and the SI Coordinator. They are trained in proactive learning and study strategies as well as facilitation skills. SI leaders attend course lectures, take notes, read all assigned materials, and conduct three to five out-of-class SI sessions a week. The SI leader is the "model student," a facilitator who helps students to integrate course content and learning strategies.
4. *Participating Students* in the SI sessions, although mentioned last, are the most crucial component of SI. The historically difficult courses frequently are introductory or "gatekeeper courses" but also include upper level undergraduate courses and courses in professional schools.

History of SI

Deanna C. Martin, University of Missouri-Kansas City, created SI in 1973. She was assigned the task of decreasing the attrition rate of minority students in the schools of medicine, pharmacy, and dentistry—and given a grant of $7,000 with which to do so. After initially offering SI at the health science professional schools and discovering success with the program, it was extended throughout the university. After a rigorous review process in 1981, the SI program became one of the few postsecondary programs to be designated by the U.S. Department of Education as an Exemplary Educational Program. The Department of Education provided federal funds for dissemination of SI until the U.S. government discontinued the National Diffusion Network.

Since its inception, faculty and staff from over 1500 institutions from twenty-nine countries have been trained to implement their own SI programs. Outside the United States, SI operates in Australia, Canada, China, Denmark, Egypt, Malaysia, Marshall Islands, Mexico, New Zealand, Puerto Rico, South Africa, Sweden, United Kingdom, and the West Indies. For example, in 1993 in Sweden, a delegation from Lund University went to visit five universities in the US. The purpose of the trip was to find a method to help students bridge the gap between studies at upper secondary school and studies at the university. They found the SI program responded to their expectations of purpose and strategy to:

- prepare students for higher education
- be a complement to higher education
- stimulate active participation
- motivate the students to be responsible for their own learning process
- develop study techniques
- gain critical thinking and the ability to solve academic problems
- collaborate in learning by working in groups

In the autumn of 1994, Lund University started the pilot SI program with two courses in mathematics. Since then, the SI program has continued to expand to other subjects at Lund University. In 1996 Lund University and the University of Missouri, Kansas City signed a contract with the common vision of spreading knowledge of, and education in the method of Supplemental Instruction and to work to further international understanding. The SI program has, since 1993, shown good results at several Universities and University Colleges in Sweden.

My experience with SI program

I learned about the program in the winter of 2008 and decided to have an SI leader in my winter term class that year. I mistakenly understood that SI leaders mostly do what student assistants do, except that an SI leader will also sit in the classroom during some lectures, tutor the students outside of the class and participate in some grading. Hence not being aware of the purpose of the program and the design of it, I regret that I did not use the program properly.

Although the program does not require an SI leader to come from the same discipline as the course content I chose an SI leader from my discipline, mathematics. She took care of all the paperwork herself and acted as the sole point of contact with the SI coordinator. I noticed that the responsibilities of the SI leader were not unlike those of a student assistant. However I did notice a few differences. One of the differences in a tutoring session and an SI session is that in tutoring sessions students usually ask questions to get help. In contrast, in the SI session, students interact with each other rather than always directing their questions and comments to the SI leader. As the program is designed to work, the SI leader would moderate a brief lectured review where students summarize a recent lesson, or identify key concepts. This review gives students a few minutes to find support in their notes and familiarize themselves with the main ideas covered during the lesson.

I believe with more contact between the instructor and the SI coordinator, the role of the SI leader would be less likely to merge into the responsibilities of a student assistant. With a clearer definition of responsibilities reinforced by constant communication between faculty and the supervisor, the program should yield greater results.

The SI leader got along wonderfully with the students who felt comfortable approaching her with questions. I had designed my class in such a way that during the second half of the class period students could work in groups and discuss possible solutions for the problems. In this format, having the SI Leader in the class helped tremendously. I had thirty-five students – which is too 'large' to ensure individual attention to all students. However, with SI Leader's help we were able to answer most of the questions. I also enjoyed having her as the grader for my class. Since she was working with the students and got to know them she was able to underline the problem areas when she was grading. Overall, I believe the impact on student learning was positive.

I had the opportunity to attend an SI workshop in Kansas City, Missouri in September 2008, which is the international center for Supplemental Instruction. I was accompanied by two of my colleagues from the department who also had an SI leader in their Spring 2008 classes. It seemed that we all had similar understanding regarding the purpose of a SI - we all thought it was another academic support model. We were surprised to discover that SI offered much more than we were led to believe.

The Kansas City workshop was very helpful and informative, and we were very impressed by what we learned about the program. We learned that the significant difference of SI over other academic support models is that SI targets high-risk courses instead of high-risk students. Since it is open to all students in high-risk courses, it does not have the stigma that is sometimes attached to the other tutoring and academic support programs. The SI promotes increased student collaboration and reinforces good study habits, which can have a positive impact on a students' overall academic performance.

By continually analyzing the effectiveness of the SI program and submitting the course reports to faculty and administration, we can help maintain and increase the level of quality for students.

I hope that I will have another opportunity to have SI leaders in my future classes. I have become a believer in the possibilities this program has to offer students, faculty and SI leaders.

References

[1] The University of Missouri-Kansas City. The International Center for Supplemental Instruction. 2007, September 2008
http://www.umkc.edu/cad/SI/overview.html

[2] Supplemental Instruction at Lund University. 1994, November 2008 http://www.si-mentor.lth.se/SI_eng/index_eng.htm

High School Mathematics Homework and WeBWorK:

A Match Whose Time Has Come.

Angelo Segalla, California State University, Long Beach

Shandy Hauk, University of Northern Colorado and WestEd, San Francisco, California

"I like a teacher who gives you something to take home to think about besides homework." --Lily Tomlin

Preface

This is the first installment of a three-part report for this journal. This first part offers a broad overview of the *WeBWorK* software and of student use. Future issues will deal with how teachers use it and what the research on web-based homework suggests about the advantages and disadvantages of using *WeBWorK*. For now suffice it to say that research has demonstrated favorable results: student achievement is higher and faster, when using *WeBWorK*, especially when teachers redirect the grading time they save to other ways of scaffolding student learning.

Homework

Mathematics homework in high school mathematics has been a problem for at least one hundred years. General Francis Walker, a Civil War veteran and later president of the Boston school board, convinced board members to curtail mathematics homework because it was making his own kids anxious.[1] More recently, Piscataway, NJ, Tampa, FL, and other communities have placed a limit on the amount of homework teachers may assign. Some districts enforce Cooper's "ten minute rule"[2] – no more than 10 minutes per day times the grade level (e.g., third graders would have 30 minutes of math homework per day). In part, these limits are driven by the knowledge that some students have more support at home than others when it comes to getting help with homework. For example, students in middle class households have a particular advantage when it comes to resources for completing homework – they are more likely to have parents who are well-educated, are at home from work in the evenings to foster homework completion, or who have the financial resources to hire a tutor.

1 General Walker

2 H. M. Cooper (2007). The battle over homework: Common ground for administrators, teachers, and parents 3rd edition. Thousand Oaks, CA: Corwin Press.

The mathematics homework pendulum, as with other issues in education, seems to take some rather wide swings over time, the reversals often coming because of national crises. Witness the Sputnik-driven changes in the 1960's (Modern Math) and the present realization—not as dramatic—that the United States is battling to maintain status as a world leader in science, technology, engineering, and mathematics (STEM).

In this article, we acknowledge the differences in beliefs and practices on homework while noting that most consider homework an important complement to the teaching-learning process. Indeed: "High school students who do their homework outperform those who do not by 69% on standardized tests." (Cooper, 2007).

The flip side of homework assignments, even temperate and well-planned ones, is the extraordinary amount of time a teacher needs (a) to determine that students actually do their homework regularly, consistently, and conscientiously, and (b) to provide feedback. In a time of full teaching schedules, full classes, in and out of class management responsibilities, and heavy state assessment accountabilities, it is worthwhile to explore alternatives to teachers spending many hours grading—or even "checking" student homework. Many teachers do not spend time on grading homework, instead using various homework checking strategies that range from "spot checking" to having students "exchange papers" in ways that may have superficial pedagogical value.

WeBWorK Overview

WeBWorK is a tireless, patient, web-based electronic homework drudge that grades every homework problem for every student in every class for the entire semester. The software also provides reports to the teacher, in spreadsheet form, on how each student performed on each assigned problem. Options in reporting include seeing the answers a student gave, number of tries made to get to those answers, and even the trail of attempts a student used. Homework can be submitted online at any time before the due date, from anywhere a student has internet access.

WeBWorK is a free, open-source, web-based software program that does not purport to teach. It is a tool through which teachers access a national library of problems, choose and assign a collection of items as homework, and harvest scoring information. The student side of the tool offers assignments to students on-screen, one item at a time (and through an optional printout of all items), collects and grades student answers, informs students whether an answer is correct, and (if this option is chosen by teachers) allows the student to try again. The tool can be used for homework, quizzes, and practice exams. Learning to use *WeBWorK*, for both teachers and students, takes less than one hour.

Students can be encouraged to work cooperatively, yet *WeBWorK* can be set to offer individualized assignments in which each student is offered slightly different problems. Research indicates that across socio-cultural and gender categories, it is common for up to 90% of students who use *WeBWorK* to do homework assignments, and to do at least as well on tests as students who do homework by the traditional paper and-pencil method. [3] [4]

Among the pedagogical advantages of *WeBWorK* are: (1) it is a free tool that delivers homework or quizzes on-line and grades and records the work for all students for the entire semester. (2) it supports the teacher by freeing time for use in instructional preparation, alternate forms of formal and informal evaluation of student understanding, etc. and (3) it is prompt in giving students immediate feedback.

WeBWorK Features

An important website, hosted by the Mathematical Association of American (MAA) where you can learn more details about WeBWorK is: **http://webwork.maa.org/moodle/**

The information on the site includes a list of the features of *WeBWorK,* key among these are[5]*:*

√ Immediate feedback as to whether the answer is correct.

√ Each homework assignment is available as a downloadable PDF file.

√ Students can use any computer and browser combination that can access a web page.

√ Every problem has a "Feedback" button that can send an e-mail message directly to the teacher

√ Students get immediate feedback on the validity of their answers and have the opportunity to correct mistakes while still thinking about the problem. As one student said, "I can fix my mistakes while [the] problem is fresh in my mind."

√ Individualized versions of problems means that teachers can encourage students to work together, while still requiring that each student develop an answer to his or her own version of the problem.

√ Provides automatic grading of assignments.

√ Provides information on the performance of individual students and the whole class.

√ The teacher can find out where individuals and the whole class are in correctly completing the homework.

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3 For example, see S. Hauk & A. Segalla (2005). Student perceptions of the web-based homework program WeBWorK in moderate enrollment college algebra courses. Journal of Computers in Mathematics and Science Teaching, 24(3), 229-253; S. Hauk, A. Segalla, & R. Powers. A comparison of web-based and paper-and-pencil homework on student performance in college algebra. Manuscript submitted for publication.

4 Rochester, Rutgers, CSULB

5 Source: http://webwork.maa.org/wiki/List_of_features

√ The teacher can use any computer and browser for management of the assignment.

√ The teacher can answer student questions by e-mail.

√ Grades from assignments are easily integrated with spreadsheets such as Excel.

√ *WeBWorK* remembers each student's problems, so they can connect to WeBWorK, attempt a problem, receive immediate feedback about the validity of their answers, try again or logout and give the problem more thought if necessary, and then reconnect to WeBWorK to attempt their own individualized problem again. Students can attempt a problem as many times as they wish until the due date unless the teacher desires to place a limit on the number of allowed attempts. Each problem in a set can have a different limit on the number of allowed attempts. For example, teachers may wish to limit the number of attempts on True/False questions while allowing unlimited attempts on problems requiring numeric and symbolic answers.

√ After the due date, students can review the homework, including the answers expected by the teacher.

How Students Use WeBWorK

After logging in much in the same way one logs into any website program, the effective use of *WeBWorK* means each student will:

1. Print a copy of the assignment.
2. Work the problems on paper, away from the computer.
3. Enter each answer into *WeBWorK* and get a validation response (correct or not).
4. If set up for multiple tries, redo any problem identified as having an incorrect answer.
5. Get immediate feedback on whether the re-try is correct or not. Note: Retries can be with or without penalty (e.g. for a quiz).
6. Turn in homework at any time, and from anywhere with web-based internet access.
7. Work in groups. Note that each student can have individualized problems, e.g., *WeBWorK* gives Tim the problem “Solve for x: $6x^2 - 13x + 5 = 0$,” while Sandy gets “Solve for x: $4x^2 - 11x + 9 = 0$”

An example

The figures below outline "Ryan" as he does Homewor Set 4 using *WeBWorK*. Perusal of the figures, even with some details left out, should give the reader a good idea of the interactive "dialogue" between WeBWorK and student. More details will be included in the second and thrd installments of this article to be published in this journal. We will examine how the teacher can monitor an individual student's and entire class' progress, concomitant implications for teaching using WeBWorK, and research results.

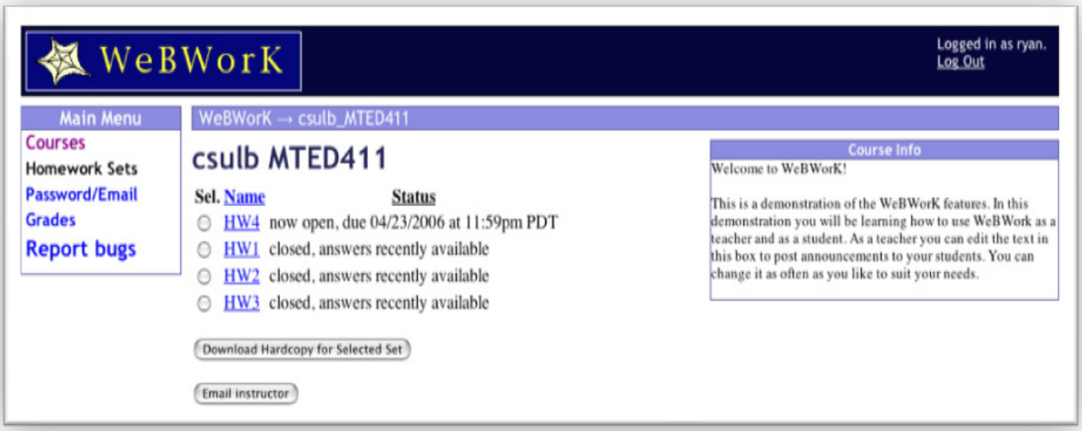

Figure 1: Today Ryan will work on HW 4. In the "Course Info" the teacher can insert directions, suggestions, and hints.

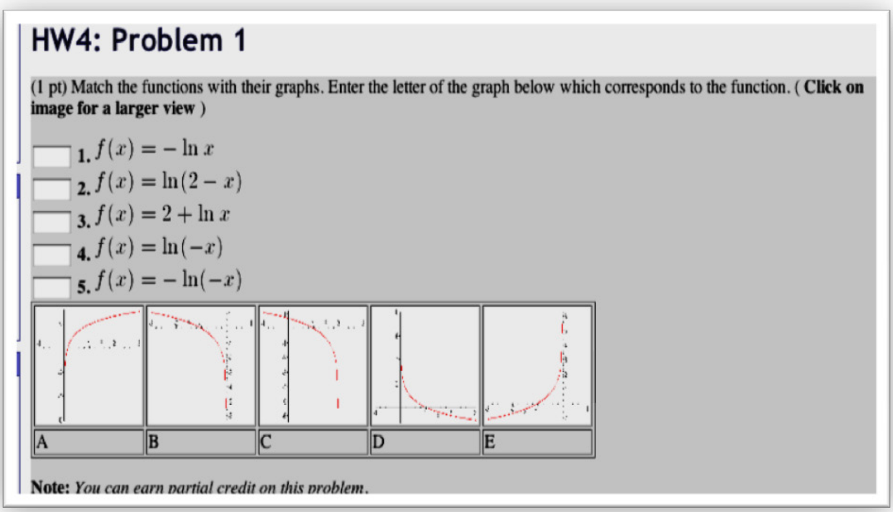

Figure 2: Ryan clicked on Problem 1 of HW 4 and got the following screen.

HW4: Problem 3

Entered	Answer Preview	Result
(P*V)/(n*T)	$\frac{PV}{nT}$	correct

The above answer is correct.

(1 pt) Solve the equation $PV = nRT$ for R.
Your answer is : (PV)/(nT)
Note: The answer is case sensitive. P, V and T are capital letters!

Preview Answers | Submit Answers

Your score was recorded.
You have attempted this problem 1 time.
You received a score of 100% for this attempt.
Your overall recorded score is 100%.
You have unlimited attempts remaining.

Email instructor

Figure 3: Ryan continued working in HW 4 and solved Problem 3 correctly. Note the summary below the problem.

HW4: Problem 4

Entered	Answer Preview	Result	Messages
2	2	incorrect	
x	x	incorrect	Your answer isn't a number (it looks like a formula that returns a number)
x	x	incorrect	Your answer isn't a number (it looks like a formula that returns a number)
2	2	incorrect	

At least one of the above answers is NOT correct.

(1 pt) Express the equation in exponential form
(a) $\ln 2 = x$ is equivalent to $e^A = B$.
A= 2
and
B= x

(b) $\ln x = 2$ is equivalent to $e^C = D$.
C= x
and
D= 2

Note: *You can earn partial credit on this problem.*

Figure 4: Ryan made an error on problem 4. Note the "Messages."

Conclusion:

That students "do" their homework regularly and consistently is an essential part of the teaching-learning process in mathematics. While *WeBWorK* is not a cure-all, it offers a valuable tool to teachers and students. It has been shown to increase the amount of time students spend on homework, and it frees up some time for the teacher to devote to other important aspects of teaching, like planning and giving feedback on more extended efforts by students to make sense of mathematics.

Common Errors from the Misuse of Order of Operations

Renuka Prakash, Delhi High School

Stephanie Souza, Merced College

Many teachers are reluctant to discuss common errors when teaching math because they are concerned that students will duplicate the errors rather than learn from them, says a recent study in *The Journal of Educational Research. "Using Errors as Springboards for Enhancing Mathematical Reasoning With Three Metacognitive Approaches," by Bracha Kramarski and Sarit Zoldan, The Journal of Educational Research, November/December 2008, pp 137-151.* This study also mentions the importance of a metacognitive culture, in which making errors is acceptable and students are encouraged to (a) self-question and analyze errors, (b) make corrections, and (c) formulate an action plan on how they have learned, what they understand and how they will remember this information.

In our combined 24 years of teaching mathematics to grades 9-14, we have found that teaching the likely errors of a particular concept - in this paper order of Operations - to students minimizes the number of errors students are likely to make. We will present common errors associated with several types of problems - exponents, simplifying algebraic expressions and formula manipulation - all stemming from the misuse of Order of Operations.

Order of Operations

First and foremost, students need to understand that, like games that we play on and off the field, the order in whch the basic arithmetic operations are carried out is a convention agreed upon by mathematicians to avoid ambiguity in deciphering expressions. A rule was defined so that evaluating or simplifying did not yield different values or answers. The rule is: Parenthesis first; Exponents second; Multiplication and Division third (performed left ot right as writen); Addition and Subtraction fourth (performed left to right as written). The most used acronym to remember this order is PEMDAS (*P*arenthesis, *E*xponents, *M*ultiplication, *D*ivision, *A*ddition, *S*ubtraction), where MD and AS occur left to right. An equally popular mnemonic to remember this order is **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally. Figure 1 is another visual to remember this rule.

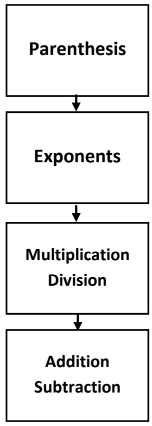

Figure 1: Sample graphic organizer for the order of operations acronym PEMDAS.

The following example demonstrates the need for the 'order of operation' convention.

Example

Simplify: $2+3\cdot5$

Common Error:

$$2+3\cdot5=5\cdot5=25$$

Notice the common error is for students to simplify the expression left to right disregarding the hierarchy of operations.

Correct Answer:

$$2+3\cdot5=2+15=17$$

Preventing this Error

The best way to avoid this error is to remind students of the acronym "PEMDAS" and emphasize that multiplication and division are performed first before addition and subtraction. Using an analogy such as putting socks on after shoes to reinforce why order matters is also very effective.

A. Exponential Expressions

Exponents, from the set of natural numbers, are shorthand for repeated multiplication of the same expression by itself. Some common errors students make include multiplying the power with the base or incorrectly evaluating expressions containing negative bases. These can be related to order of operations.

Example 1

Simplify: 2^3

Common Error:

$2^3 = 2 \cdot 3 = 6$

Correct Answer:

$2^3 = (2)(2)(2) = 8$

Preventing this Error

As per order of operations, exponents must be performed first, not multiplication. In this case, the exponent 3 means multiply 2 with itself 3 times.

Example 2

Simplify: (a) $(-2)^4$ (b) -2^4

Common Error:

Typically students ignore the parenthesis due to their lack of understanding of their importance as a grouping symbol. Common answers are: (a) –16 and (b) 16.

Correct Answer:

(a) $(-2)^4 = (-2)(-2)(-2)(-2) = 16$

(b) $-2^4 = -(2 \cdot 2 \cdot 2 \cdot 2) = -16$

Preventing this Error

The error in the above example could be avoided if the importance of parenthesis is emphasized at the beginning of the topic of exponents. For part (a) students do not realize that $(-2)^4$ means $(-2)(-2)(-2)(-2)$. For part (b) the common mistake is that students simplify the expression the same way as part (a). A good technique would be to have students write -2^4 as $-1 \cdot 2^4$ just as $-x$ can be written as $-1x$. Teaching example (a) and (b) together helps students to differentiate the placement of negative signs.

B. Simplifying Expressions

When simplifying expressions, the idea of "cancelling out" is misconstrued by most students. Factoring is key to simplifying the following examples, but is often forgotten. In these examples, *x* and *y* are elements of the real number system.

Example 1

Simplify: (a) $\dfrac{3x^3 - x}{x}$ (b) $\dfrac{x^2 - y^2}{x - y}$

Common Error:

One common, incorrect simplification is $\dfrac{3x^3 - \cancel{x}}{\cancel{x}} = 3x^3 - 1$ where students "cancel out" the x term neglecting the x in the $3x^3$ term. In part (b), students tend to cross cancel x and y in the denominator with x and y in the numerator and write the answer as $x - y$.

Correct Answer:

(a) $\dfrac{3x^3 - x}{x} = \dfrac{x(3x^2 - 1)}{x} = 3x^2 - 1$

(b) $\dfrac{x^2 - y^2}{x - y} = \dfrac{(x + y)(x - y)}{x - y} = x + y$

Preventing this Error

Students need to be clear that a fraction bar is a symbol of grouping or inclusion. In part (a) it is actually $3x^3$-x multiplied by $1/x$. Order of operations demands that paranthesis be done before division. It is the same concept in part (b) where the problem must be understood as x^2 - y^2 divided by x - y. Previewing and reviewing factorization techniques will help prevent this error. Also, a simple example such as $\dfrac{2(3)}{2} = \dfrac{\cancel{2}(3)}{\cancel{2}} = 3$ versus $\dfrac{2+3}{2} = \dfrac{5}{2}$ not 3, can help to bring to light the power of addition and subtraction when there is a division bar. Students should see that the addition or subtraction of terms in the numerator and denominator prevents cancelling terms unless factoring takes place. One technique is to begin teaching the above concept with simple expressions such as $\dfrac{x+8}{2}$ explaining that

$$\frac{x+8}{2} = \frac{x}{2} + \frac{8}{2}.$$

Example 2

Here is another classic example of an expression that cannot be simplified, but students will sometims try to regardless of the rules.

Simplify: $\sqrt{x^2 + y^2}$

Common Error:

$$\sqrt{x^2 + y^2} = \sqrt{x^2} + \sqrt{y^2} = x + y$$

Correct Answer:

$\sqrt{x^2 + y^2}$ cannot be simplified further.

Preventing this Error

First observe that the given expression is another way of writing $(x^2 + y^2)^{1/2}$. Applying order of operations, requires students to evaluate or simplify expressions in the paranthesis first. However, the two unlike expressions cannot be added.

An alternate way of helping students with the scenario above is to encourage them to put values for x and y and check if both sides of the equation are equivalent. For example, when we let $x = 4$ and $y = 3$ we can write $\sqrt{4^2 + 3^2} \stackrel{?}{=} \sqrt{4^2} + \sqrt{3^2}$. When students simplify both sides they will obtain 5 = 7 which is a false statement. This is a good opportunity to have students write the rules as they think it should be, such as $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$ along with numerical examples of the rules. When a numerical example is applied, the students are forced to ask why and what the correct rule is. They can then prove to themselves how it is the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ works when theirs, $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$, did not. It is important for the students to understand and differentiate between the two operations.

C. Formula Manipulation

Students deal with formulas not only in math but also in other subjects. Substituting the values in a given formula and doing formulaic calculations often confuses students. The quadratic formula is a prime example.

Example

Use the quadratic formula, $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, to solve the equation:

$x^2-6x+8=0$

Common Error: $x=\frac{-6\pm\sqrt{-6^2-4\cdot1\cdot8}}{2\cdot1}=\frac{-6\pm\sqrt{-36-32}}{2}=\frac{-6\pm\sqrt{-68}}{2}$

Students place some values correctly in the formula, but sometimes forget the negative sign like the -6 in this example. In this case, order of operations is also performed incorrectly.

Correct Answer: $x=\frac{-(-6)\pm\sqrt{(-6)^2-(4\cdot1\cdot8)}}{2\cdot1}=\frac{6\pm\sqrt{36-32}}{2}=\frac{6\pm\sqrt{4}}{2}$ which gives us $x=4$ or $x=2$

Preventing this error

A closer look at this error will take the students back to the example on exponents where the students failed to see the difference between $(-2)^4$ and -2^4. Placing the a, b and c terms in parenthesis such as $x=\frac{-(b)\pm\sqrt{(b)^2-(4ac)}}{2a}$ can help students make fewer mistakes with order of operations and exponents by narrowing their focus on the computations that are to be done first. This could be done for any formula. Reviewing order of operations and rules of exponents at the beginning of this topic is a good refresher to avoid mistakes.

Conclusion

All students bring informal mathematical knowledge and problem solving capacities to class. The mistakes students make will provide learning opportunities for everyone. Teachers can use knowledge of common student thinking to anticipate errors and pose questions to help students review a concept. The errors discussed above are very common and constantly repeated as students progress into more complex mathematical concepts.

A little background knowledge and review is necessary for the students before they are taught the heart of a concept. This helps students to avoid making basic mistakes. In my experiences, discussing (common) errors routinely while teaching exposes students to the type of errors they are likely to make, while enriching their mathematical skills. They will know the likely pitfalls and will try to avoid them.

Some of the methods to analyze and minimize errors that were discussed were graphic organizers, writing mnemonics, use of ELMO or overhead to display student's errors, colorful visual-aids and formulaic manipulations. Graphic organizers are intended to provide a visual method of the best way to remember information. It is a very helpful tool and can be used to self-question, analyze errors, scaffold, and prepare action plans to understand information better, i.e., to encourage metacognitive approach to math learning. Use of mnemonics is a strategy where students design their own way of learning and remembering concepts, and become less dependent on teacher designed mnemonics. ELMO is great technology and can be used to display student work without names to generate a discussion on the type of errors made along with how to avoid them. Discussing errors is a good technique to gauge the capabilities of your class and prepare instruction according to the needs of the students.

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Mathematical Modeling Using Difference Equations

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Abstract : A mathematical model uses mathematical language to describe a natural phenomenon and is used to solve real-life problems. One mathematical technique used for modeling is the difference equation. The goal of this paper is to demonstrate mathematical modeling problems using difference equations. In a brief introduction one type of difference equation will be demonstrated. Then the method solving this difference equation will be shown. Finally, several real-life mathematical modeling problems and the procedure for solving these problems will be demonstrated.

1. Introduction

A mathematical model is a mathematical representation of a process, device, or concept by means of a number of variables which are defined to represent inputs, outputs, internal states of the device or process, and a set of equations and inequalities describing the interaction of these variables [1]. One mathematical technique used for the modeling procedure is the difference equation.

It is common in real-life application problems that we may not be able to describe the situation with an explicit formula for the terms of a sequence. Instead we may know only some relationship between the various terms. An equation which expresses a value of a sequence as a function of the other terms in the sequence is called a difference equation. In particular, an equation which expresses the value of a sequence y_n as a function of the term y_{n-1} is called a first-order difference equation [2], where $n = 1,2,3,\cdots$. Here y_0 is an initial condition. A first-order difference equation can be expressed in the following way [3] :

Definition [Difference Equation] :

$y_n = ay_{n-1} + b$, where a and b are real-valued parameters. This paper is organized in the following way: In section 2, the method to solve the difference equation will be demonstrated. Real-life mathematical modeling application examples using the difference equation will be shown in section 3. Finally the conclusion will be followed in section 4.

2. Solutions for the First Order Difference Equation (without formal proof)

In this section, the method of solving the first order difference equation will be introduced [3].

From the general form of the difference equation $\boldsymbol{y_k = ay_{k-1} + b}$, where a, b are real numbers, we get

$$y_1 = ay_0 + b$$

$$y_2 = ay_1 + b = a(ay_0 + b) + b = a^2 y_0 + ab + b$$

$$y_3 = ay_2 + b = a(a^2 y_0 + ab + b) + b = a^3 y_0 + a^2 b + ab + b$$

In this way, we will obtain a general form for $\boldsymbol{y_n}$ for $k = n$:

(1) $$y_n = a^n y_0 + a^{n-1} b + a^{n-2} b + \cdots + a^2 b + ab + b$$

We consider the following two cases to solve equation (1) in terms of $\boldsymbol{a}$, $\boldsymbol{b}$, and an initial condition $\boldsymbol{y_0}$.

Case I: $a \neq 1$

Multiply both sides of (1) by $\boldsymbol{a}$, then subtract the new equation from (1).

$$ay_n = a^{n+1} y_0 + a^n b + a^{n-1} b + a^{n-2} b + \cdots + a^2 b + ab$$

Then we obtain

$$y_n - ay_n = a^n y_0 - a^{n+1} y_0 - a^n b + b$$ which implies

(2) $$(1-a) y_n = (1-a) a^n y_0 - ba^n + b$$

Now, divide both sides of the equation (2) by $(1-a)$, since $a \neq 1$.

Then $$y_n = y_0 a^n - \frac{b}{1-a} \cdot a^n + \frac{b}{1-a} = \frac{b}{1-a} + \left(y_0 - \frac{b}{1-a}\right) a^n.$$

(3) Therefore, $$y_n = \frac{b}{1-a} + \left(y_0 - \frac{b}{1-a}\right) a^n$$

which is of the form $y_n = A + Ba^n$

Case II: $a = 1$

Set $a = 1$ in equation (1). Then

$$y_n = a^n y_0 + a^{n-1} b + a^{n-2} b + \cdots + a^2 b + ab + b$$
$$= y_0 + b + b + \cdots + b + b + b = y_0 + bn \; .$$

(4) Hence, $y_n = y_0 + bn$

3. Mathematical Modeling Application Examples Using Difference Equations

Example 1 [Finding the Amount of a Mortgage] :

Mortgages can be described by difference equations, as follows. Let y_n be the unpaid balance on the mortgage after n months. In particular, y_0 is the initial amount borrowed. Let i denote the monthly interest rate and R the monthly mortgage payment. Then

[balance after ***n*** months] = [balance after ***n-1*** months]+[interest for month]-[payment]

$$y_n = y_{n-1} + iy_{n-1} - R$$

(5) $y_n = (1 + i) y_{n-1} - R$

Question : Suppose that you can afford to pay $978.90 per month and the yearly interest rate is 7.5% compounded monthly. Exactly how much can you borrow if the mortgage is to be paid off in 30 years?

Answer : The monthly interest rate is $\frac{.075}{12} = .00625$.
Hence, $\mathbf{y_n = 1.00625 y_{n-1} - 978.90}$ from the equation (5). This means $\mathbf{a = 1.00625}$ and $\mathbf{b = -978.90}$. Moreover, $\mathbf{y_{360} = 0}$, since 30 years are equal to 360 months.
Then we obtain equation (3)
$\mathbf{0 = 156{,}624 + (y_0 - 156{,}624)(1.00625)^{360}}$.
Solving for y_0, we get $140,000. Therefore, the initial amount of $140,000 can be borrowed (see figure 1).

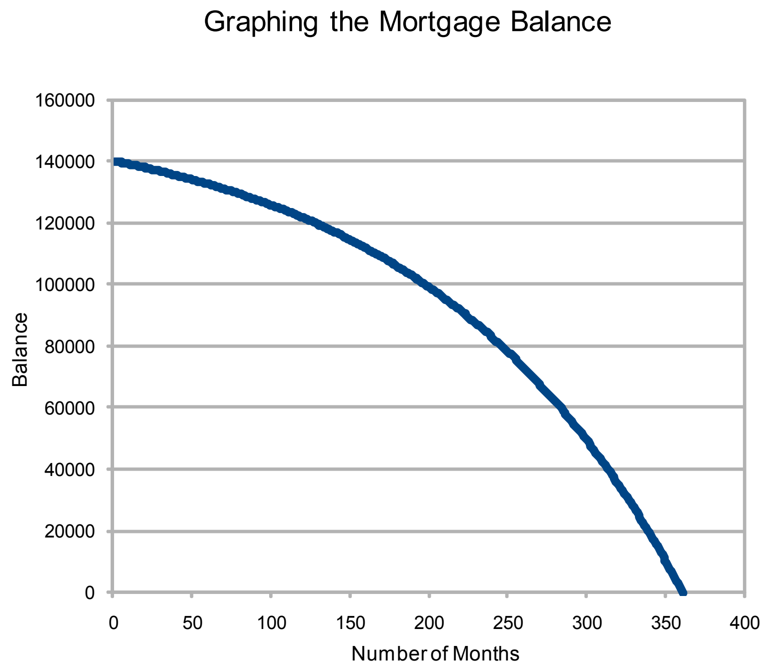

Figure 1. Graph of mortgage problem with initial amount borrowed of $140,000.

Example 2 [Growth of a Bacteria Culture] :

A bacteria culture grows in such a way that each hour the increase in the number of bacteria in the culture is proportional to the total number present at the beginning of the hour. Then the difference equation describing the growth of a bacteria culture can be modeled in the following way :

Let y_n be the number of bacteria present after n hours. Then the increase from the previous hour is ky_{n-1}, where k is a positive constant of proportionality. Therefore,

[number after n hours]=[number after n-1 hours]+[increasing during nth hour]

$$y_n = y_{n-1} + ky_{n-1}$$

(6) $y_n = (1+k)\, y_{n-1}$

Question : Suppose that the constant of proportionality is 0.05. Assume that initial number of bacteria was 5,000 and find the number of bacteria after 15 hours.

Answer : Since k is 0.05, we have $y_n = 1.05 y_{n-1}$ in equation (6). Hence $a = 1.05$ and $b = 0$. Here $n = 15$. Plug these values into equation (3). Then we have

$y_{15} = 5{,}000(1.05)^{15} = 10{,}394.6 \approx 10{,}395$. Therefore, the number of bacteria after 15 hours are 10395 (see figure 2).

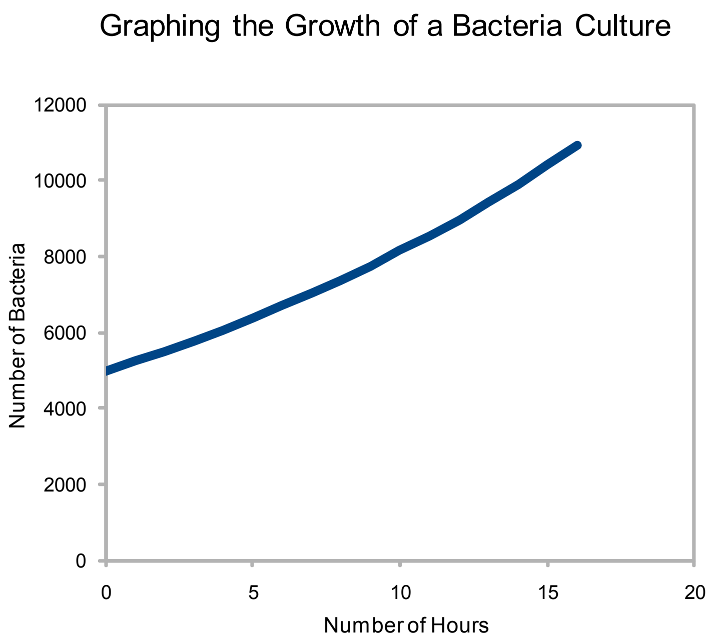

Figure 2. Graph of growth of a bacteria culture with initial population of 5,000.

Example 3 [Spread of Information] :

Suppose that at 8 A.M. on a Saturday the local radio and TV stations in a town start broadcasting a certain piece of news. The number of people learning the news each hour is proportional to the number who had not yet heard it by the end of the preceding hour. Then the difference equation describing the spread of the news through the population of the town can be modeled in the following way :

Let y_n be the number of people who have heard the news after n hours and P be the total population of the town. In addition, let k be a positive constant of proportionality. Then the number of people who have not heard the news after $n-1$ hours is $P - y_{n-1}$. Therefore,

[number who knows after n hours]=

[number who knows after $n-1$ hours]+[number who learn during nth hour]

(7) $y_n = y_{n-1} + k(P - y_{n-1})$

Question : Suppose that the constant of proportionality is 0.3 and the population of the town is 50,000. Find the number of people who have heard the news after 3 hours.

Answer : Since $\boldsymbol{k}$ is 0.3 and $\boldsymbol{P}$ is 50,000, equation (7) becomes $\boldsymbol{y_n = 0.7y_{n-1} + 15{,}000}$. This means $\boldsymbol{a = 0.7}$ and $\boldsymbol{b = 15{,}000}$. Here $\boldsymbol{y_0 = 0}$. Plug these values with $\boldsymbol{n = 3}$ into equation (3), then we obtain $y_3 = 50{,}000 - 50{,}000 \cdot (0.7)^3 = 32850$.

Therefore, 32850 of people have heard the news after 3 hours (see figure 3).

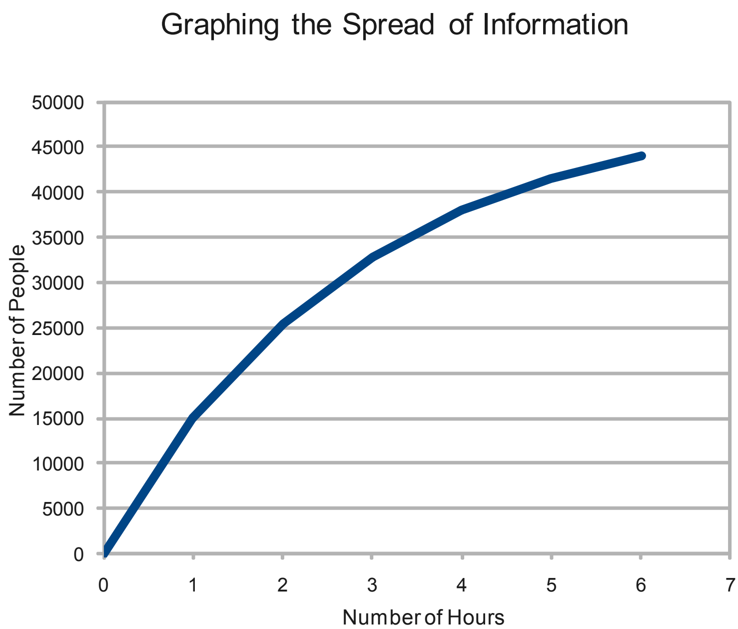

Figure 3. Graph of spread of information problem with initial population of zero.

4. Conclusions

The goal of this paper has been to demonstrate mathematical modeling problems using the difference equation, $\boldsymbol{y_n = ay_{n-1} + b}$. In this paper, a particular first-order difference equation is introduced. Difference equations are widely used to describe natural phenomenon. Three specific examples, including finding the amount of a mortgage, growth of a bacteria culture, and spread of information were shown to demonstrate to use of difference equations in the mathematical modeling process.

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It’s Never Too Late to Teach Old Dogs - or Teachers - New Tricks!

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“Retirement at sixty-five is ridiculous. When I was sixty-five I still had pimples.”
George Burns

Ridiculous indeed! In fact, the authors of this article have nearly ninety years of combined teaching experience in the mathematics classroom. The theme of this short article is that we both discovered a new tool—“trick” simply better fits in the title’s idiomatic phrase—rather late in our career, and we are both thrilled about this find. Called WeBWorK, this tool, if used as an integral part of a mathematics course, can boost student learning and at the same time free the instructor from the necessary but laborious chore of grading homework. Pimples or not, we claim -and earn- sensitivity to what may or may not work in the mathematics classroom; these two old timers will tell you WeBWorK works!

What is WeBWorK?

First, a word about WeBWorK: sponsored by the National Science Foundation, it is a free, open-source, web-based program for delivering, grading, and recording, individually different—but conceptually the same—mathematics homework problems to a dozen or “four score” students, or in a large group instruction of one or two hundred students.

WeBWorK is the brainchild of mathematics professors Michael Gage and Arnold Pizer of the Department of Mathematics at the University of Rochester. With assistance from the National Science Foundation the WeBWorK development team was expanded to include a half-dozen mathematicians/programmers from five universities around the country. Presently, WeBWorK is supported and enhanced by many other national and international colleges and universities where WeBWorK is used as an “electronic assistant” for mostly lower division mathematics homework and quizzes.

In keeping with the quintessence of our title we only peruse WeBWorK here. For a more in depth exploration, for the nuts and bolts of WeBWorK, the reader should begin with the MAA website:
http://webwork.maa.org/moodle/.
From there, one will be directed to more in depth sites, contact people, discussion boards, and all the way to the programmers who maintain and embellish the program continually; the latest improvement is the addition of Java applets to the program.

Benefits of WeBWorK to Instructors

We found that especially for large classes WeBWorK is a blessing. The program, as mentioned above, reports to the instructor, via spreadsheet, who did the homework, who did not do the homework (always nice to know!), how many problems were done, how many tries it took to get the problem correct—or not—and even when and how long the student worked on the homework. (Human nature being what it is, it is not unusual to find some students posting their homework at 11:59 PM on the day the homework is due!) In short, when the instructor looks at the spreadsheet she can tell who, what, when, and how, the problems were done. If, say, problem 17 shows a large number of retries, the instructor had better address the concept behind that problem in class at the next meeting,

Every student gets the same "template" of a problem with parameters that are randomized by the student ID as the randomization seed. For example, one student may be asked to find the definite integral of a quadratic polynomial function with a particular set of coefficients and limits of integration, another student will be asked the same question but will be given a different set of parameters. This discourages copying since each student must provide his or her own unique answer.

WeBWorK allows the instructor to log in anytime to check the progress of the entire class or an individual student. The instructor can easily set or re-set the deadlines for the entire class or an individual who may need extra time or a different set of problems. It is easy to spot students who do not do homework or do just a portion of it and perhaps then it is time for the instructor to call the student in to "talk things over." There could be, and often there are, in today's busy not-school-related student lives, valid reasons for not completing a homework assignment and special arrangements can be made if the instructor so chooses. We also take into account the fact that WeBWorK allows for partial credit, but this partial credit only reflects the percent of the problem a student got correct; thus if she answers correctly four out of five on any given problem WeBWorK will show 80% for that problem.

Speaking as teachers, our favorite WeBWorK feature is that WeBWorK provides students with immediate feedback as to the correctness of their answers and allows them to make multiple attempts (with no penalty, if the instructor so chooses) until they (hopefully) succeed in solving the problem.

Benefits of WeBWorK to Students

The program is accessible via the Internet using any browser, so students can do their homework at anytime and from wherever they have Internet access. For those in which internet is not affordable or available, students could be allowed to do the homework on paper and enter their work using one of the school's computers. WeBWorK also provides immediate feedback, allowing students to correct their mistakes while they are still working on the problem. In class, we encourage students to keep their textbook and notes next to them and consult these resources as they do the homework on-line. We also encourage them to work together (each student gets a different set of paramterters for the same problem; to seek help from classmates, or tutors, or the math lab personnel, or the instructor (the last via email built into WeBWorK) or during office hours. At times getting an incorrect response repeatedly may be syntactical, not a mathematical problem. [9]

How do the students like WeBWorK? Our collective and informal experience shows that "most" students like it. Each student can monitor his or her own progress. They like the immediate feedback feature. Secondly, they like being able to correct their errors in privacy and without penalty. We recommend that to avoid the pressure of being in front of a computer monitor while doing their homework, students should first print out their (unique) problem set in pdf format (quickly available in WeBWorK) and do the homework away from the computer and in a quiet place. Afterward, they can enter their answers-and check them-by using WeBWorK. Another advantage of WeBWorK: the student cannot "just hand in" the homework, with right or wrong answers. She knows when an answer is correct or not.

Advantages of WeBWorK

Research shows students who use WeBWorK for mathematics homework do as well as and in many cases better than students who use traditional paper-and-pencil homework. [2,7,9] Some subpopulations tend to do better (based on GPA) when using WeBWorK. [7] That is research also shows that students who tended to get lower grades in previous mathematics courses get better grades when using WeBWorK. Some researchers claim that this is so because the program keeps students on task longer; they do not "give up" as quickly if they cannot get a right answer. [10]

Finally, we need to emphasize that WeBWorK is not what most people think of as Computer Assisted Instruction (CAI). The program does not teach. Rather, it delivers, presents, grades, and provides immediate feedback on assigned homework problems and keeps track of student responses.

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Twelve Challenging Problems

Improving problem solving techniques

Francisco Garcia, Los Baños High School

CONCEPT: Measurements and Geometry; Mathematical Reasoning; Algebra and Functions.

SKILLS: Using critical thinking and logical reasoning for problem solving.

GRADES: 7-12

MATERIALS: Pencil, binder paper, graph paper and ruler.

DESCRIPTION: Twelve Challenging Problems are designed to sharpen students' problem solving skills. These exercises will help students review graphing, order of operations, angle measurements, how to work with radical expressions and use formulae for the area of polygons and perimeter of polygons. These may be used as Problem of the Week (POW).

DIRECTIONS:

1. The teacher will write down one of the Challenging Problems on the whiteboard and/or chalkboard every Monday.

2. Students will work on the problems throughout the week, and have their solutions ready by Friday of the same week.

3. Teacher and student discussion will take place on Fridays to discuss the solutions to each week's Challenging Problem.

PROBLEMS:

1. The surface of a 3-inch cube is painted, and then the cube is cut into 27 one-inch cubes. How many of the one-inch cubes are painted on one side? Two sides? Three sides? How many cubes have no paint? (Build model.)

2. The recipe for a cake in a square pan measuring 8” × 8” × 2” needs 2 eggs. The baker
has to bake a large birthday cake using this recipe. The baker wants to use a pan with
dimensions 3 times that of the one in the original recipe. How many eggs will be
required, if the baker wants to fill the new pan?

3. The same rule is applied to each input to arrive at the outputs in Table 1 shown. For this linear relationship, what will the output be when you input 31?

Input	Output
-4	2
0	14
3	23
8	38
18	68
31	?

Table 1. Set of inputs and outputs after same mathematical operations are performed.

4. Find the exact value of $\sqrt{8+2\sqrt{7}}-\sqrt{8-2\sqrt{7}}$.

5. Charlie is 3 times as old as Joel, and Joel is 10 years older than Michael. If we know that the sum of all three ages is 80, how old are each of the three men?

6. There are 20 tiles of three different colors in a bag. The number of blue tiles is twice the number of red tiles. There are 2 green tiles. Find the number of tiles of each color that are in the bag.

7. James was asked to add 4 to a given number and then multiply the result by 3. However, he added 3 to the given number, and then multiplied by 4, getting an answer of 60. Follow the instructions correctly and find the correct answer.

8. Solve for n: $n\overline{)367}$ with quotient 25^{R17}. Note: 367 = 25n + 17.

9. Find the area of a convex pentagon with vertices (-2,0), (-3,4), (-1,7), (5,8), and (8,2).

10. A man walks into a donut shop and buys one-half of the donuts that are on the shelf. The next customer buys two-fifths of the remaining donuts. The next customer buys six donuts and the last customer buys two-thirds of the remaining donuts. In the end, 15 donuts remain on the shelf. How many donuts were on the shelf at the beginning? (Hint: solve by working backwards.)

11. Without using a calculator, find out which is smaller 5^{3000} or 3^{5000}.

12. In numbering the pages of a textbook, a total of 825 digits were printed. If the first
page is numbered 1, how many pages does the textbook have?

SOLUTIONS:

1. There are 3 cubes on each edge of the original cube. Each cube at the center of each side of the cube has paint on one side. There are 6 sides so we have 6 cubes with paint on one side. There are 12 edges on a cube. We only consider the cube in the middle of each edge. So the number of cubes painted on two sides is 12(1) or 12. The cubes on each corner of the cube have paint on three sides. This gives us 8 cubes with paint on three sides. Last, there is one cube at the center of the cube with no paint on any side.

2. The dimensions of the pan increase its volume (V) by a factor of 27. The new volume of the pan in *(3l) (3w) (3h) = 27V*. The ingredients must be increased by the same factor of 27, so the number of eggs we need is $27(3) = 54$.

3. Find the difference of each subsequent input and the corresponding subsequent outputs. From top to bottom, notice that when we multiply each input difference by 3, we get the corresponding difference of the output. That is, for the first two rows 4 x 3 = 12. We add the first entry of the output (2) and the first difference of the output (12) and we get the second entry of the output. This relationship will hold for all input/output pairs in this table of values. To find the output when the input is 31, we multiply the difference 31 – 18 by 3, and we get 39. Then we add 68 + 39 and we get 107.

4. First, let $x = \sqrt{8+2\sqrt{7}} - \sqrt{8-2\sqrt{7}}$.

   Second, square both sides to get $x^2 = \left(\sqrt{8+2\sqrt{7}} - \sqrt{8-2\sqrt{7}}\right)^2$.

Third, use distribution to get $x^2 = 8 + 2\sqrt{7} - 2\sqrt{(8 + 2\sqrt{7})(8 - 2\sqrt{7})} + 8 - 2\sqrt{7}$
Fourth, simplify to get $x^2 = 16 - 2\sqrt{64 - 4 * 7}$. Next, we have $x^2 = 16 - 2\sqrt{36}$
Simplify the right side of the equation to get $x^2 = 4$. Now we take the square root of both sides to get two possible solutions $x = \pm 2$. We only keep $x = 2$ because it is the only value of x that will give us a true statement for the original problem.

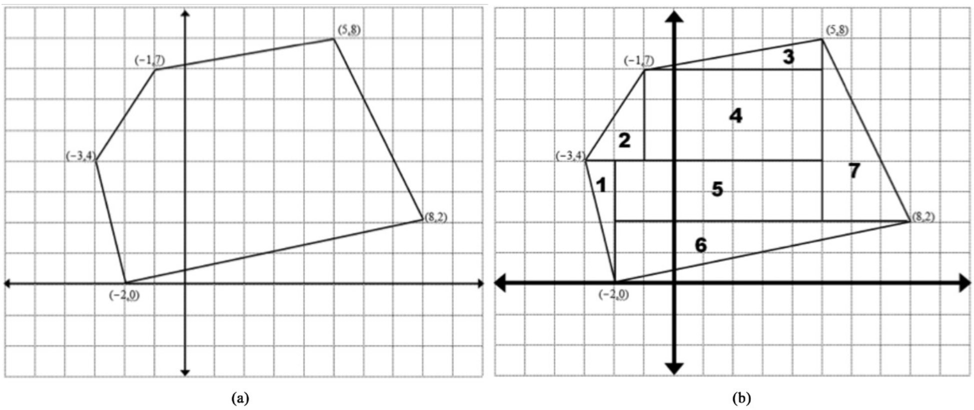
5. Let c = Charlie's age, let j = Joel's age and let m = Michael's age. Charlie is 3 times as old as Joel, so we have $c = 3j$. Joel is 10 years younger than Michael, so we have $m = j - 10$. We know that the sum of the ages is 80. We can write the equation $c + j + m = 80$. Substitute $c = 3j$ and $m = j - 10$ into $c + j + m = 80$ and get $3j + j + j - 10 = 80$. Solve the latter equation to get $5j = 90$ for j, then $j = 18$, Joel's age. Then substitute the value of j into $c = 3j$ and $m = j - 10$ to find that Charlie is $3(18)$ or 54 years old and Michaels age is 8 years old.

6. Let b= the number of blue tiles, let g = the number of green tiles and let r = the number of red tiles. There are 20 total tiles, so we have the equation $b + g + r = 20$. The number of blue tiles is twice the number of red tiles, so $b = 2r$. There are 2 green tiles, so $g = 2$. Substitute the values $b = 2r$ and $g = 2$ into the equation $b + g + r = 20$ to get $2r + 2 + r = 20$. We find $r = 6$. We have $2(6)$ or 12 blue tiles, 6 red tiles and 2 green tiles.

7. Find x with the incorrect instructions. So we have to solve the equation $(x + 3)4 = 60$ for x. We get $x = 12$. Next, follow the instructions correctly to compute $(x + 4)3$. Plug in $x = 12$, and get $(12 + 4)3$ or 48.

8. Set up the equation $25 + \frac{17}{n} = \frac{367}{n}$ or $25n + 17 = 367$. Then, solve the equation for n and get $n = 14$.

9. Draw a point for each ordered pair on the coordinate plane. Next, connect each point to create the convex pentagon (see figure 1a). Draw vertical and horizontal segments to divide the convex pentagon into 5 triangles and 2 rectangles (see figure 1b). Label each triangle and rectangle using numbers 1 through 7. The area of triangle 1 is 0.5(1)(4) or 2 units. The area of triangle 2 is 0.5(2)(3) or 3 units. The area of triangle 3 is 0.5(6)(1) or 3 units. The area of rectangle 4 is (3)(6) or 18 units. The area of rectangle 5 is (7)(2) or 14 units. The area of triangle 6 is 0.5(2)(10) or 10 units. The area of triangle 7 is 0.5(3)(6) or 9 units. The total area is 2 + 3 + 3 + 18 + 14 + 10 + 9 or 59 units.



(a) (b)

Figure 1: (a) Convex pentagon drawn on the Cartesian plane. (b) Solution of area of convex pentagon divided up into triangles and rectangles.

10. In the end there are 15 donuts remaining. We follow the computations backwards by doing the opposite of each computation in each step. First, multiply 15 and 3 to get 45 donuts. Second, add 45 +6 to get 51 donuts. Third, we know that 51 is 3/5 of the next number we need to find. One-third is equal to 17 donuts, so 5/3 is equal to 17 x 5 or 85 donuts. Last, we multiply 85 x 2 and we get 170. Thus, we had 170 donuts to start.

11. Consider the original numbers 5^{3000} and 3^{5000}. Apply properties of exponents to rewrite the original numbers and get, $(5^3)^{1000}$ and $(3^5)^{1000}$. Then, evaluate expressions in parenthesis to get $(125)^{1000}$ and $(243)^{1000}$. Since $125 < 243$, we have $125^{1000} < 243^{1000}$ So, 5^{3000} is smaller.

12. Pages 1-9 require a total of 9 digits; pages 10-99 (exactly $99-10+1=90$ two-digit pages) require a total of $90*2$ or 180 digits. Pages 100-199 ($299-200+1=100$ three digit pages) require $100*3$ or 300 digits. Then pages 200-299 ($299-200+1=100$ three digit pages) require 300 more digits, giving us a total of 789 digits. We still have 36 digits to account for and we know that each page requires three digits, so we have $\left(36/3\right)$ or 12 pages. Thus, the number of pages in the textbook is $9+90+100+100+12$ or 311.

A Creative Way to Review the Multiplication Table

Stella Estrada, Wakefield Elementary

CONCEPTS: Number Sense, Mathematical Reasoning

SKILLS: Memorize Multiplication Table, Recall of basic multiplication facts

MATERIALS: Activity Sheets 1, 2 and 3 for teachers. No sheets for students. Activity Sheet 1 with the list of songs associated with each set of multiplication facts. Activity Sheet 2 with details on how the songs and facts are linked. Activity Sheet 3 with facts listed in a format that they link with the song.

BACKGROUND: From the very beginning of my teaching career I knew that teaching *all* students would be a big challenge. I also knew that, as a fifth grade teacher, my greatest challenge would be to find creative ways to review the multiplication table. Our school district is located in an area where most of the students are English Language Learners. These students, not having the English Language foundation and the lack of academic language, need several exposures in mastering the multiplication facts. I wanted to find a modality that would enable my students to learn and recall basic multiplication facts.

This article describes one successful method I use to re-teach and review the multiplication table. Studies show that teaching through music is effective and that just about everybody loves music. A colleague and I put our heads together and decided that, maybe, if we connect the multiplication table to music, it would be livelier, more catching, and would help recall.

We researched into the most commonly learned songs during a child's first eight years of life, and associated them to each one of the multiplication facts. This strategy proved effective because students, at this level, had some knowledge of the multiplication facts. The method gave them a hook to recall facts by association with familiar songs. The students accepted music as a way to learn the multiplication table.

After a few years of experimenting with different songs, we were successful in compiling the list of tunes described in this article. The best part of the final outcome is that the tunes may be adapted, changed or revised to fit a teacher's style. English Learners enjoy these songs because it is a 'whole class choral activity.' A student need not be taken out of the homeroom class or pulled out to go to a "special class" to review or be re-taught this information.

PREPARATION OF MATERIALS: Activity Sheet 3 facts must be copied onto poster boards – one poster board for each fact - large enough for the class to see. It is not necessary for the students to have or see Activity Sheets 1 and 2. The intent of the activity is for all students to learn and memorize these so as to facilitate recall. Preparing these posters takes time but once done, it can be used for years to come.

Handwritten posters, each fact with a different color helps in the spontaneous of recall, as students tend to link patterns and colors in memorizing.

DIRECTIONS:

- Students form a circle.
- Mostly students know the facts for 5 and 10. It is best to begin by chanting these in rhythm as indicated in activity sheet 1. This allows them time to learn to be in sync with the class.
- Next pick a number whose number facts need to be learned
- Then pick the song corresponding to that number from Activity sheet 1. The teacher leads the students by singing the first 2 lyrics of the original song.
- Teacher and students sing the original song together. Repeat this until all students are familiar enough with the song and are able to sing-a-long with the teacher with ease.
- The teacher and students sing the first half of multiplication facts (see activity sheet #2) to the selected tune for the multiplication facts. Repeat verse by verse.
- Repeat in chunks until song is learned.
- Repeat steps 1, 2, 3, and 4 for each of the activities/songs.
- While students are singing they may apply a different activity/body language that is asked by the teacher.

In short, the directions boil down to, “I do, we do, they do.”

- The teacher leads the students in the song/activity.
- The teacher and the students sing together/while-doing activity.
- Students sing in chorus while doing the activity.

SUGGESTIONS: Teacher may want to add gestures and other physical activities/ body movements while singing the songs. Here are some suggestions:

- Hold appropriate finger up that show how many times the number is multiplied
- Tap in palm of hand with appropriate number of fingers
- Do a wiggly dance with the song
- Use hand motions that connect to the song (Wheels on the bus: open hands at your sides and do the motion of a train going down the track).
- Snap your fingers, skip, hop or jump as grade appropriate!

The author would like to thank Dr. Viji Sundar for her guidance in the writing of this paper.

Activity Sheet 1

List of Songs and the "Number" associated with it.

Song	Number
If you're happy and you know it	2
This land is your land …..	3
Ol' MacDonald had a farm …	4
Chant tapping index and middle finger in palm of opposite hand	5
The wheels on the bus go round and round (making circular motion with open palms at sides)	6
Happy Birthday …..	7
She'll be comin' round the mountain …..	8
Oh say can you see …..	9
Chant tapping index and middle finger in palm of opposite hand	10

Activity Sheet 2

2's tune: If You're Happy and You Know It
If you're happy and you know it clap your hands (clap, clap)
2 4 6 8 10 (clap, clap)
If You're happy and you know it clap your hands (clap, clap)
12 14 16 18 20 (clap, clap)

3's tune: This Land Is Your Land
This land is your land, this land is my land
3 6 9 12 15 18
From California to the New York Islands
21 24 27 30

4's tune: Ol' MacDonald
Ol' MacDonald had a farm, eee ay eee ay O
4 8 12 16 20 24
And on his farm he had a duck
28 32 36 40

5's: CHANT

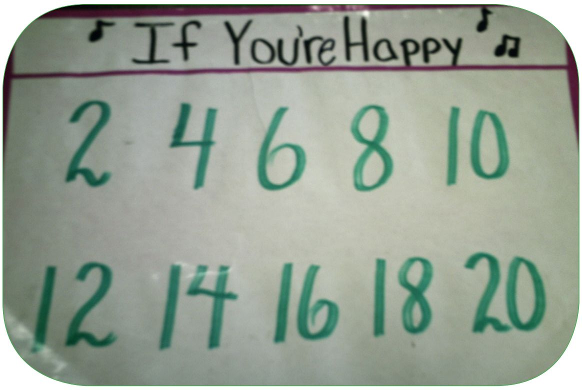
6’s tune: The Wheels On the Bus Go Round and Round
The wheels on the bus go round and round
6 12 18 24 30
Round and round, round and round
36 42 48 54 60

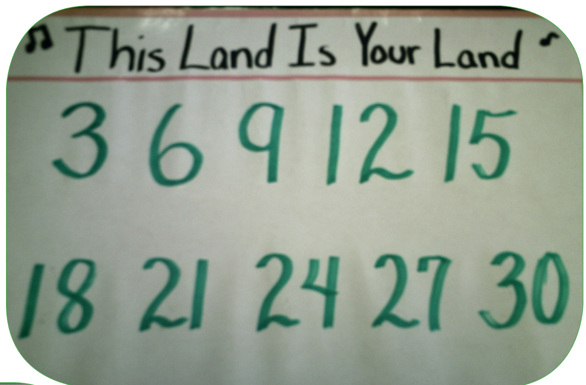
7’s tune: Happy Birthday
Happy Birthday to you
7 14 21
Happy Birthday to you
28 35
Happy Birthday Ms. Estrada
42 49
Happy Birthday to you
56 63
And seventyyyyyyyy

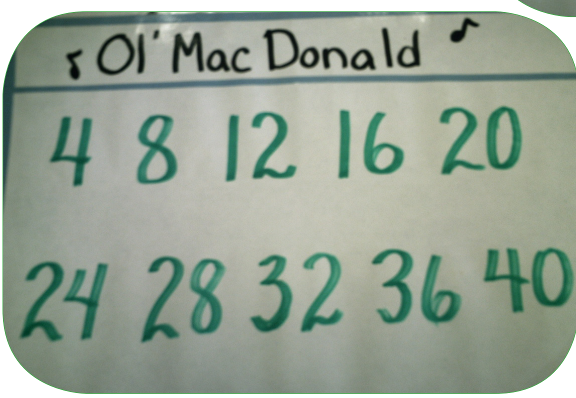
8’s tune: She’ll Be Commin’ Round the Mountain
She’ll be commin’ round the mountain when she comes
8 16 24 32 40!
She’ll be commin’ round the mountain when she comes
48 56 64 72 80!

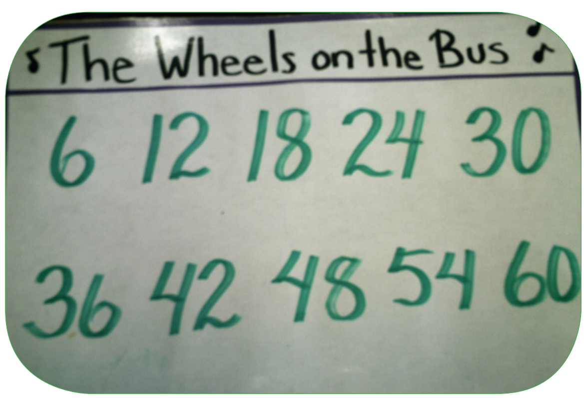
9’s tune: Oh Say Can You See!
Oh, say can you see
9 18 27
By the dawns’ early light
36 45
What so proudly we hail
54 63
By the twilight’s last gleaming
72 81 90

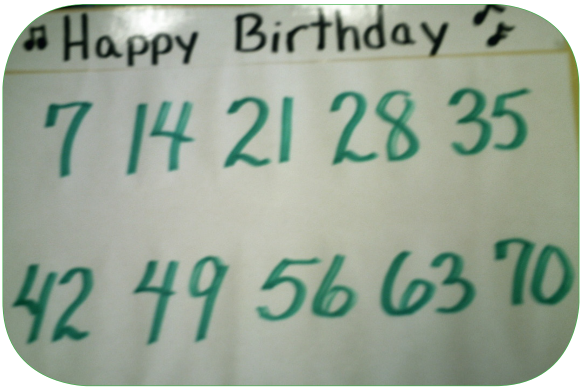
Activity Sheet 3

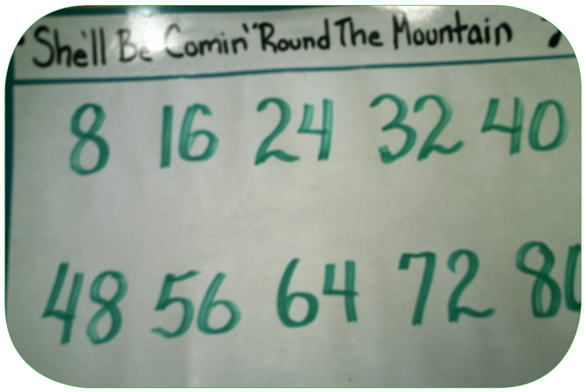


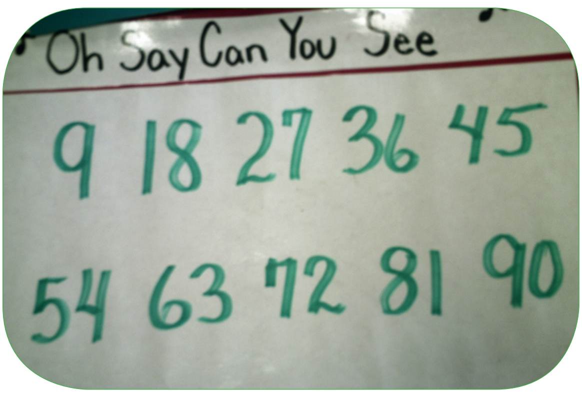







Happy Birthday
7 14 21 28 35
42 49 56 63 70


She'll Be Comin' Round The Mountain
8 16 24 32 40
48 56 64 72


Oh Say Can You See
9 18 27 36 45
54 63 72 81 90

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