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Background. The *Journal of the California Mathematics Project* (*JCMP*) is a publication of the California Mathematics Project (CMP) and is sponsored by San Francisco State University. The journal supplements the official news publication of the CMP, *California Online Mathematics Education Times* (COMET), published at ⟨https://cmpso.org/comet/⟩. The journal publishes a wide array of submissions, including brief research and research-to-practice articles, reports of classroom practice, and book reviews. We also welcome reviews of state adopted materials and insights about programs from authors who have experience with them.

Equity, Joy, and Professional Learning in Mathematics Education. This issue of the *Journal of the California Mathematics Project* illuminates how California educators, researchers, and teacher developers are cultivating more just, humanizing, and joyful mathematics classrooms. Across these contributions, three themes emerge. Student success characterized by equity remains central, from Brown's framework advocacy to Pilgrim et al.'s exploration of instructors' beliefs and Ichinose's and Stone-Johnstone and colleague's structural innovations for equitable learning. Together, they reveal that achieving equity in mathematics requires more than fair access. It demands transformation in mindset, systems, and pedagogy.

A second theme in this issue is the emotional and human dimension of teaching. Melnick's work foregrounds affect and empathy, while Pilgrim and colleagues document how immigrant instructors' personal histories shape classroom equity efforts. Brown and Melnick also emphasize teacher agency and courage to act differently, echoing a broader call for educators to "feel mathematics" and foster joy as a route to justice.

Finally, professional learning and systemic reform link the issue historically and institutionally. From the apprenticeship model (Ichinose) and flipped-classroom study (Stone-Johnstone et al.) to Jones's historical review of Wilson's book, *California Dreaming*, each piece highlights how teacher preparation, institutional design, and policy must align to sustain reform. The recurring lesson, voiced in Wilson's 1990s history and carrying through Brown's 2020s reflections, is that equity cannot thrive without robust, ongoing professional support and a willingness to rethink entrenched structures.

Together, the articles in this issue portray California mathematics education as a living laboratory: reflective, critical, and persistently hopeful about what equitable mathematics teaching can become.

Submission and Review of Material for Publication. The journal's mission continues to be communication about mathematics education among those engaged in it, including those active in the CMP or similar initiatives anywhere. Contributions to *JCMP* are made by K-12 teachers, higher education faculty, and a variety of others involved with research and development in mathematics education, such as graduate students and school leaders. The call for submissions is on-going. We do accept simultaneous submissions (copyright is retained by the author). If *JCMP* is the first to accept an article for publication, then the *JCMP* publication must be cited in all other publications of that article, even in revised form.

Manuscripts are accepted in .rtf, .doc/.docx, and .tex formats, using 12 point Times New Roman or a font with similar size and spacing and with 1.25 inch margins on all sides. Articles are published with LaTeX(a production editor works with authors on formatting). List references at

the end of the article in alphabetical order by first author last name with appropriate corresponding citations in the text in American Psychological Association (APA) 7 style. See the articles in this volume for examples of appropriate style and length. For more information and the electronic submission website, please see:
`https://jcmp.calstate.edu`

Submissions will be refereed and may be accepted or returned for revision. Initially, we use a double blind consideration process. To help prepare an article for publication, a referee may reach out to the author to suggest improvements.

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Interview with Kyndall Brown on Equity and the California Mathematics Framework

Matthew G. Jones

Abstract. On July 12, 2023, the California State Board of Education approved the 2023 Mathematics Framework for California Public Schools. Kyndall Brown, Director of the California Mathematics Project, played a role in the development of the framework, and has long been an advocate for greater equity in school mathematics. This interview was conducted in August, 2023.

MJ: I thought maybe a good place to start is around equity, because, you know, everyone talks about equity, but then they're not always talking about the same thing. So I thought maybe something in the way of a definition or some examples of what equity looks like in the classroom.

KB: The definition that I like the most is the one that talks about not being able to predict academic outcomes based upon race. So that if I were to walk into the most advanced math class at any school, it should reflect the demographics of the school.

I also think about, you know, equity of opportunity, making sure that all students have access to qualified teachers who are strong in their content and pedagogy, and having access to all of the resources that they need to be successful. So, just removing all barriers so everyone has an equal opportunity to be successful.

What that might look like in a classroom is, students working collaboratively in heterogeneous groups, engaging in low-floor, high-ceiling tasks, engaging in discourse and dialogue using whatever instructional tools, whether it's technology, manipulatives, just having the access to whatever it is they need to be successful.

MJ: Okay, so pivoting off of that, what do you see as the role of the California framework in promoting equity?

KB: So, I think the framework really took a stand around equity, unlike what I think has been done in previous frameworks. They have specific chapters dedicated to equity. Chapter 2 is focused on teaching for equity and engagement, and then chapter 9 is about structuring school experiences for equity and engagement. The framework has really taken a very bold and unequivocal stance that equity is important for the teaching and learning and mathematics.

MJ: Yeah, and then kind of pivoting off that, what influence do you think the framework ends up having in classroom instruction? I don't know, there's a lot of history of wrangling over the framework and then sometimes it feels like you go into classrooms and what the class looks like before and after the framework doesn't seem all that different.

KB: Yeah, and you know that's a challenge right because I question whether or not most teachers really take the time to go in depth with the framework. Most teachers tend to simply use whatever curriculum that they have to deliver instruction. So I think that it depends on what kind of investment schools and districts are making in professional learning, whether or not teachers are then reaching out to, say, the offerings of an organization like the math project or going to conferences like the California Mathematics Council (CMC) or National Council of Teachers of Mathematics (NCTM) where they're going to learn more about the framework. I think it's really dependent on those kinds of things. I think I think the more enthusiastic teachers are gonna seek it out for themselves and find ways.

I think, again, there are going to be the other ones who just receive whatever professional learning that their district or school offers and then I think for other people there, they may not change at all. They may just stick to what they've been doing in their textbook. Unless they really follow their textbook closely and it's whether their textbooks have really made changes to align with the framework.

MJ: Right, and then it's a question of whether they have the tools to make that happen the way the textbook has in mind.

So, with the journal readership being primarily teacher educators and people in leadership positions, how do you see teacher educators and professional learning providers supporting teachers, either around the framework and equity or just around equity in general?

KB: One of the things that I like about the framework are those chapters I was referring to. In chapter 2, they talk about those 5 components of equitable and engaging teaching for students. They talk about teaching around big ideas, using open-ended tasks, teaching towards social justice, inviting student questions and conjectures, and then prioritizing reasoning and justification. And so to me, those are very concrete and practical things that teachers can learn how to do. It is going to really depend on what kind of professional support that teachers have to engage in those different practices that the framework presented. I think that's gonna be a huge piece of it that's going to really make or break whether or not the framework is implemented is the kind of professional learning teachers have.

MJ: Yeah. And do you think there's a robust enough network out there in terms of the support? Do you think there's the capacity to do that kind of training just to make that happen?

KB: I think if there is the proper investment now it's interesting because the folks at the California Department of Education have indicated that there really is no money for a framework and there really was none for the 2013 framework. And so those of us in the math education community have been really talking about how we then support something like this. One of the examples that we have to look towards is what the science folks did in terms of rolling out the Next Generation Science Standards. The California Science Project partnered with the county offices of education and the statewide science organization and designed rollouts for the standards, and these were 2-day rollouts that were held at county offices and they did them across the state. They've rolled it out in phases. They realize that once was not enough, that there was so much complexity to the standards that they needed to do continual rollouts and phases and then things that I think they're still engaging in.

I think that's the kind of approach that we're gonna have to take as well. I think we probably need to target district leadership so that we can make sure that they're well versed. Then they can go back and support teachers within their districts. We need to offer it to anybody who wants to come, teachers, administrators, or coaches, to the training, to get a more in-depth understanding of what the framework is asking for.

And to me, to be honest, we have to kind of backtrack and really do a good rollout of the 2013 framework that never happened. There are still lots of teachers across the state who aren't really well versed in the Standards of Mathematical Practice. We're gonna have to catch up with that and then add on with what this new framework is asking teachers to do.

MJ: So, I thought maybe we pivot slightly and talk about tracking as a piece that gets discussed a lot around equity. And I thought there's this idea of having an accelerated track that some people really latch onto. Do you see acceleration and equity as competing priorities or is there a way for those to work together or complement each other in some way?

KB: Well, it's thinking about, first of all, why are we accelerating, and when do we accelerate? I think those are two important things.

I don't know that there is as much need to accelerate in middle school, given the caveat that I really think that the middle school standards from the 2013 framework are really strong. They're very rigorous, very robust. And if they're taught well, they can really support a student's strong mathematical development. I think we always have to pay attention to students in their varying levels of development and you may have some students that are more advanced, but I think that there are ways to engage students in the standards in middle school without accelerating. I think that this whole idea of low-floor, high-ceiling tasks allows more advanced students to engage at a higher level. I think there's a lot of really rich curriculum.

I will tell you what else is a challenge with the 2013 framework: the content of the eighth grade course was changed so that the second semester is what has traditionally been the first semester of an algebra one course. So that ninth grade Algebra 1 course is a lot more robust but it doesn't really cover all of that stuff from the first semester, all the linear functions and all that. That's not covered anymore in the ninth grade algebra course. And so, to me, I don't really know what's being taught in algebra in middle school, whether they're teaching the old traditional algebra one course, right, which is the first semester with most of the linear stuff, the second semester all the quadratic and exponential stuff, or if they're teaching that ninth grade Common Core Algebra 1 course. If they are, then our students are missing some of those really important eighth grade standards that they should be getting. I think that usually the goal of acceleration for the most part is to get students to Calculus before they graduate high school. I just think there's a lot of ways to do that that don't require acceleration necessarily at the middle school. I think it can be delayed until high school. You just gotta be creative. I see that for some students there might be a need to give them more advanced content, but I don't think there's a need to try to rush them through that college preparatory math sequence before they're ready. And I really think there's only a small percentage of students who really need to be. I think that a lot of people are using this whole idea of acceleration as parents of privilege to make sure that their children are on that higher math track whether or not they are really advanced. I just think there are a lot of issues that are involved with this whole idea of acceleration.

I don't know if admissions counselors are gonna have to start looking at things differently or what, because there are a lot of people that are taking calculus only because it's gonna give them a better chance of getting into their school of choice, not on majoring in any STEM field or anything like that. And so then to me then they're not actually getting what they need. They might be much better served by statistics or data science or discrete math or a lot of these other alternative courses, but you know, they're kind of forced into taking calculus, cause that's what colleges recognize as rigor.

MJ: Yeah, and then I guess the other thing is there's acceleration and then there's also theoretically, a separate thing, honors or an enriched class, which could be a class that does more instead of just goes faster, right? But I don't think that really exists most of the time. I think most of the time it's, let's just do it faster.

KB: Exactly, right. I agree. There's so much more that can be done. Think about how much of the curriculum doesn't get covered traditionally that you could do, and just different mathematical topics that you could expose students to besides just algebra and geometry. There are so many opportunities and statistics always gets short shrift. I just think we could turn on so many more students if we were able to get to those statistics and probability chapters that are at the back of the text. Move them up to the beginning of the semester or the beginning of the school year. I think we could get a lot more students.

MJ: Yeah, yeah. Certainly, there's a lot to be done with data that could be very engaging for students, and probably doesn't get–even with the increasing focus on some of these specific courses in data science–the attention that it could, or attention proportional to how much of it is in those standards at the earlier grade levels.

What do you see in terms of these, you know, at the high school level, the different pathways? What is their function or how do you think of it in some ideal sense if they were implemented? In your vision, what would be different pathways for students, or who would benefit from being in one pathway versus another?

KB: Well, you know, I think all the research shows that students benefit when they take 4 years of college preparatory mathematics and unfortunately, like I say, their options in many cases are just so limited. I think by offering alternative pathways you probably get more students taking that fourth year of math and they'd be able to learn different types of mathematics, and different ways in which they could use mathematics to further their own educational or career options.

I think it's just really exposure to different ways that math is relevant. I mean, if a student isn't gonna be a STEM major, it's hard for them. It's hard for teachers to convince them that using factoring in quadratic equations is going to be useful for them in some kind of way, whereas I think some of the things that happen in the statistics course or the discrete math course or even a modeling course, students will be able to see the relevance of it a lot more and it might be a motivator for them to continue on taking STEM courses in the future. Because what we do now is say you'll need to use this at some point in your life later and it's just not a convincing argument.

I'm actually working with a Gates funded project where they're trying to come up with narratives to help students see the utility of math. They've been doing these focus groups and they've been interviewing students, teachers, and parents. For the most part, students only see the utility of basic arithmetic. They don't see the utility of anything beyond. They don't see the use for algebra, geometry, anything like that. This is true even when talking with their parents. The parents express the same beliefs that they're really not gonna be using algebra and geometry, they only use basic math in everyday life. What the Gates Foundation is trying to do is to craft arguments that would help convince students. What doesn't convince them is you're gonna use it later in life. That means nothing to them. All these things that we as teachers always use, it's like that carries no weight with them at all. What are gonna be some examples or things that we can do? Because even if you tell them, we're doing some particular function today and this is how you're gonna use it in your life, students react like, whatever. So, what are gonna be these arguments that we're going to use to convince students that they're gonna use and need math later in their lives? This is an interesting project.

MJ: Yeah. Or it makes me think of the studies, that they must be 30 or 40 years old at this point, but the ones where kids that work with their parents as street vendors were doing all kinds of math and then they would go to the classroom and use none of that when it came to doing the same arithmetic.

KB: Right, the Brazilian candy. Yeah, that's a good example. And yeah, how many times are people doing that in their regular daily life?

MJ: Right, right.

KB: Well, we even talk about students' perseverance, but if it's something that they're interested in, like, how many times will they fail at a video game that they're interested in, and they fail and start over because it's something that they're motivated to do right. So, it's not like they're not capable of persevering. I don't know if it's possible to make math have that kind of interest, and I've seen teachers motivate students to want to engage, but it's not easily done.

MJ: Right, right, yeah. I kinda wonder, what are some of the levers that we could be pulling that would make it easier? There are always a select few teachers that are able to pull it off, but en masse, what does it really take to make that kind of thing happen? Whenever you find these studies of a whole school that's transformed, like think of the ones that Jo Boaler's worked with, whatever they get in place lasts for a few years and then somehow the wheels come off and it's back to the same old thing 6 years later or whatever.

KB: Yeah, yeah, and I think about the Railside school that was kind of undermined by the California standards and all that madness. It almost takes somebody who's willing to buck the system and not follow procedures and do things out of the box. That's how you're gonna see the kind of changes when you're not about pacing plans and periodic assessments and all that kind of stuff, and not feeling pressured by those kinds of things if you're really gonna do what we're talking about. It takes a certain kind of agency on a teacher's part if you really want to implement whatever it is with fidelity, whether it's You Cubed or Illustrative Math or whatever. If you're really gonna stick to the way that they want you to do it then you have to say, "Well, okay, I'm not gonna pay attention to this pacing plan, I'm gonna veer here when I need to."

MJ: Yeah. Well, that needs an ongoing effort towards what we sometimes call buy-in. There may be better terms for it, but making sure that all the stakeholders are willing to cut you the slack to try something that doesn't look like what they thought math instruction was supposed to look like.

KB: Right, and you know, you brought up a whole other issue that I hadn't thought about. There was this article that came out back in the 1990s about how elite parents undermine reform. It's in the Phi Delta Kappan, but I even think about it just like you say even though people have really bad experiences with mathematics, it's almost like they want their children to have that same bad experience.

I can remember being a parent and going to my child's school parent meeting, and this one parent is saying, "The teachers don't teach math, they just put the students in groups and they tell them to figure out the problems on their own, and when the students ask for help, they tell them to go ask their group members. I want these teachers to teach, right?" Now luckily I was there to kind of break it down, and I said, "Well, no, this is what they're trying to do." But that's the pushback. We're really trying to do it differently. We don't want your children to have the same bad experience you have, but then it's like you want them to have that same [experience].

I don't know if you're familiar with Liljedahl's work on building thinking classrooms.

There is a Facebook group that I've been paying attention to and teachers who said I'm getting all this push back from parents because of what I'm trying to do, and pushback is coming from parents and students. So, how do you navigate all that while you're trying to do things differently? If we want things to change, then we're gonna have to do something different. How do you manage all that complexity?

There just has to be a parental educational component to this, where we inform the parents of what we're doing and why, and what they can expect to see, so it's not such a shock to them.

You know, the common core is the butt of all these jokes all the time on social media or you hear, "Must be that common core math because I don't get it." We definitely need to do a better job in terms of PR, what it is that we're trying to do and why.

I don't know if you've read Heather McGee's book, *The Sum of Us*, but she talks about the zero sum thinking that permeates the US, whether it's economics, education, politics. She starts by giving the example of how in Arkansas during the civil rights movement, after the passage of Brown versus Board of Education, they desegregated everything. Rather than allow African-Americans to swim at the city pool, they cemented it in, so now nobody gets to swim, and so who benefited from that, right? .

So much of what happens is the zero sum thinking, and it's really pervasive in math ed especially. Why can't we have all students take calculus? [Instead of] one class that we filter everybody out, why can't we offer multiple sections of calculus so everybody could take it?

MJ: Yeah. Alright. This has been great.

KB: Take care.

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Bringing Joy to Uninspired Teachers of Math
Touchstone Strategies, Part 3†

Hal Melnick

Abstract. The third in a three-part series, this resource describes and illustrates three of eight Touchstone Strategies for teacher educators to use in their work with mathematics teachers. The article explores how to inspire teachers to find the joy in mathematics so they can support their students to do the same. Through a variety of tools, techniques, and helpful hints, the eight touchstone strategies in the series illustrate what high quality mathematics instruction looks like and how teachers can reframe their own thinking about mathematics to create deeper learning opportunities for their students. This piece, Part 3, describes the three touchstone strategies: *work a problem "to death," non-dominant language*, and *concept teaching games*.

Touchstone Strategy #6: Work a Problem "To Death"

Work one intentionally perplexing problem "to death" unearthing confusions that arise.

I have found it beneficial to provide teachers repeated opportunities to deeply solve a confounding problem in a collaborative group, in order for them to see that it can and should take time. Each semester, I would give people two to three classes to work on what I called "The Letricia Problem" (mentioned in earlier parts of this article and described below) — probably the most eye-opening task I have ever watched teachers work on together. During those three weeks, we intersperse concept games related to measuring and geometry, which are topics embedded in the Letricia Problem. In addition, we do some Number Talks and have a few text-based discussions on articles about the conceptual roots to the geometric and measurement ideas embedded in the problem.

If teachers actually live through a productive and enlightening problem-solving process themselves, I believe they are more likely to become courageous enough to offer the same eye-opening experience for their own students. They tell me "doing" math like this in my class is their linchpin since most rarely have done that before enrolling in graduate school.

After reading and discussing the chapters in Making Sense (Hiebert et al., 2000), particularly the ones about the chapter focusing on the social culture of a progressive math classroom, they are more than primed for this experience. They read and we discuss how a group of four girls in

† From an original report by Melnick (2018), this is the third of a three-part reprint with the permission of the author. The full report is available from the Bank Street College of Education website.

fourth grade struggle to communicate when solving a division of fractions problem (Using 20 apples, how many apple tarts can be made if it takes $\frac{3}{4}$ of an apple to make just one tart?) After listening to kids working through their different perceptions and conjectures about solutions, the class is ready to mirror the same process.

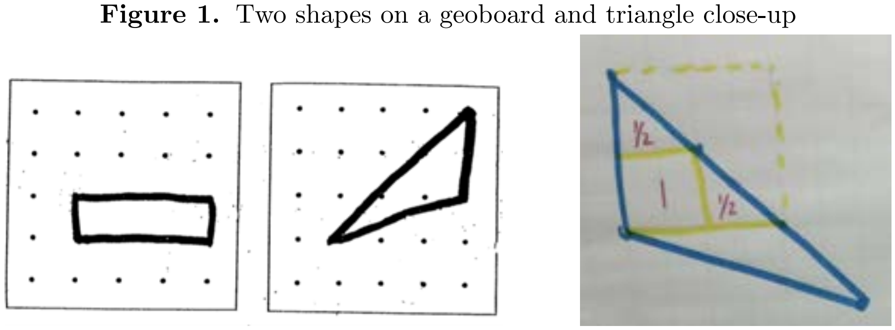
My work with collaborative groups starts with forming groups with people of varied skills and needs (re: style of communicating, feelings about math, potential learning disabilities, varied pace of learning needs, as well as the age they teach). Have them solve a complex problem by showing at least two different strategies, including one that uses concrete tools while abiding by Burns' "Three Rules for Small-Group Work" (see Touchstone Strategy #3 in Part 2). This is the time for us to "walk the talk." I teach as I expect them to teach. This is the moment for theory to intersect practice. We learn by doing. They need to collaborate and I need to stay out of their way.

This Perplexing Math Problem solving event is an opportunity for them to practice the skills they are developing as math teachers: listening, giving hints to others without giving answers away, honoring different styles, different paces, not being overbearing in pushing an agenda on others, being open to new and different ways to think about the whole problem you are working on, being interactive. It is an opportunity for each of them to be a good teacher to one another. I insist upon multiple representations, which was one of the five NCTM's Process Standards in NCTM's Curriculum and Evaluation Standards for School Mathematics (1989). I tell them to use drawings, arrows, equations, words, and cutouts depicting their visual thoughts. I do so with the express purpose of enhancing equity and accessibility. I model one way to undercut overvaluing the so-called 'brightest' or 'fastest' learners who could otherwise try to monopolize class time and brag about their erudite solutions.

The genesis of the key problem I use, called the "Letricia Problem," came about when I was working with doubting teachers of grades three to five in one public school where I was consulting. They each claimed that they had shelves filled with wooden geoboards in their classrooms sitting there for years and years, but no one saw a use for them. They also doubted my suggestions that heterogeneous cooperative learning groups could magnify learning for all and could possibly satisfy children's needs across varied levels of sophistication. They challenged me to prove to them that the geoboard could be used to achieve the goals I had put forth for math learning, including those embedded in the state standards for their grades.

On the spot, I made two shapes on one geoboard. One was a 3 x 1 rectangle and one was a scalene triangle. See Figure 1.

Figure 1. Two shapes on a geoboard and triangle close-up



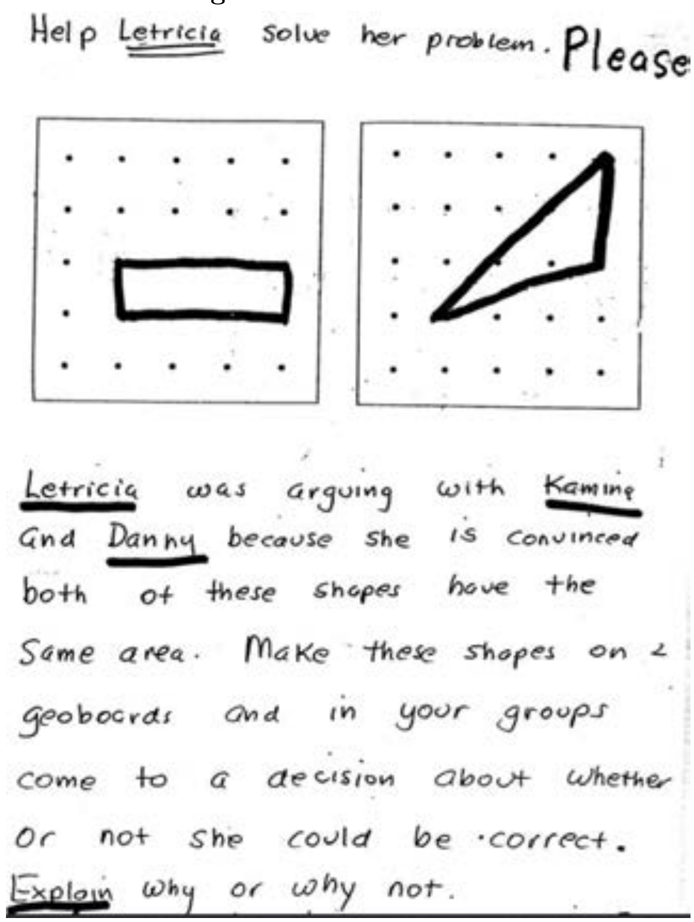
I thought they were about the same size in area but I could not at first tell. I knew that it would be easy to find the number of square units in the rectangle, but saw in front of me that the triangle's area would be a bit harder to figure out in square units. I myself did not immediately know how to do it. By using rubber bands I could see there was an easy part to figure out with an area of two square units inside it, but then there was an annoying leftover skinny triangular section that would require some manipulation and use of logic in order to find its area.

I decided to go with these two shapes. I had no idea how many different methods could be used to solve it, but I trusted it could be solved somehow. I went with this in the moment and presented the problem using a classic method I had been taught by Bob Davis when I was a young teacher in the 1970s studying his Madison Project Paradigm Teaching Strategy at Syracuse University (Davis, 1980).

I made up a girl named Letricia and imagined her in a class with two boys called Kamina and Danny. Using Davis' Paradigm Teaching Strategy, I created a hypothetical situation where three kids were arguing or debating the relative merits of a solution or a strategy to a problem. The premise to Davis' strategy is then to ask the class to resolve the children's hypothetical conflict by thinking together using words, numbers, drawings, or equations and, in at least two ways, write, draw, and define your group's solution. This worked like a charm and has been replicated in every one of my 80 or so classes since then.

Figure 2 is the Letricia Problem as presented to students in the class.

Figure 2. Letricia Problem



Below are the strategies that emerged during one section of the Spring 2017 class. I always require at least two strategies. I tell them, by having two irrefutable strategies, they can verify they are right. I also ask people to name the strategies they employed using their own nonconventional language. We post the name of their preferred strategy next to their explanations. On the next pages are images of the strategies that emerged during my last semester of teaching. I have done this problem for at least 20 years of classes and it seems fresh to me each and every time. Geoboards, brightly colored dot paper, scissors, glue, bands, markers, and large poster paper are all provided. Teachers first solve the problem quietly as best they can alone, then they share their conjectures with their group and begin to work together in class. After considerable time to listen to each other and to delegate responsibilities for the final product presentation, they represent their findings using at least two methods.

To help me assess each student's ability to work in a group, I also require a multimedia Perplexing Problem assignment where groups of four present their solutions to one perplexing problem done during the semester. They may choose to represent the Letricia Problem but they also have a list of other challenging ones from which to choose. They are to compose a presentation and write it up together. After designing their PowerPoint or Google Slides presentation, they are each to write: 1) what they learned about math by working collaboratively and 2) what they learned about themselves as a member of a collaborative math group.

The following are some of the insightful responses I feel depict many of the emotional challenges teachers encounter when doing collaborative math together. Read the following with a lens toward the emotional affect. What feelings or emotions are evoked when Bank Street students are asked to reflect on the active process of working within a group whose end assignment they had to submit as a shared entity? Is there any evidence of coping and adaptation? Might they be learning from what they feel after going through this?

> I learned that everyone needs to go at their own pace. Once I got the answer, I wanted to be done, or at least have everyone thinking and feeling the same way as I did, but I realized that some people needed more and it was my job as a team member to give that time. I also learned that problem-solving in a group is REALLY rewarding! I felt like by the end of the time we had really gotten to know each other as learners and teachers. —Abby

> I feel I was being impatient at times. Is it harder to be on the inside as one of the group members or on the outside as the teacher? I felt I needed to keep switching back and forth between the two mindsets. The process is rewarding and challenges you to really listen. Definitely a lesson in patience. Some of the qualities that make us caring adult group members have come with maturity and can't be fully expected from kids who are not developmentally capable. —Alex

> As a member of a collaborative group, I found myself to be patient in the process. This is because I myself am very curious as to how others in my group solve problems. Further, proving that I am someone who builds huge amounts of confidence from having others present in the collaborative process. In fact, because math in general can be a stressful time for me as a learner, having others present makes the process that much easier, making math an engaging experience. —Gabe

> I learned that it is hard for me to deal with the disequilibrium caused by exposure to different approaches and lines of thinking. I experienced frustration at my inability to get team members to take on my perspective. I discovered that my team members were open to letting me go in a different direction, explore my own curiosities about the problem, and share back what I had discovered. —Merry

Each semester I wonder about how long I can devote to one problem. And each semester I am deliriously happy that I have allowed as much time as I did. Not only do I feel I unleash wildly varied thinking in my students, but also a range of mathematical content. We don't wake up each day and do number operations first or geometry at 10:40 AM or algebra at 1:30 PM. When you solve a real life problem, you use all the knowledge you have, and you ask friends to help you because they might have some bits of information and skills you don't have. That is what 'doing math' should look like and the Letricia Problem offers just that. Just doing a quick scan at the photos of the sample from one class, we see how the math discussed included:

(1) discussion of polygonal characteristics,
(2) doing area calculations using formulas remembered or invented,
(3) using doubling and halving,
(4) spatial logical reasoning,
(5) the Pythagorean theorem and exponential work,
(6) trigonometric work with opposite angles, and finally
(7) aspects of high school motion geometry using "flips, slides, and turns" otherwise called reflections, translations, and rotations respectively.

All this is possible with just one problem. Creative problem solving and strategic competence is developed as well. How much better can a class get?

Figure 3. Sample work on the Letricia problem.

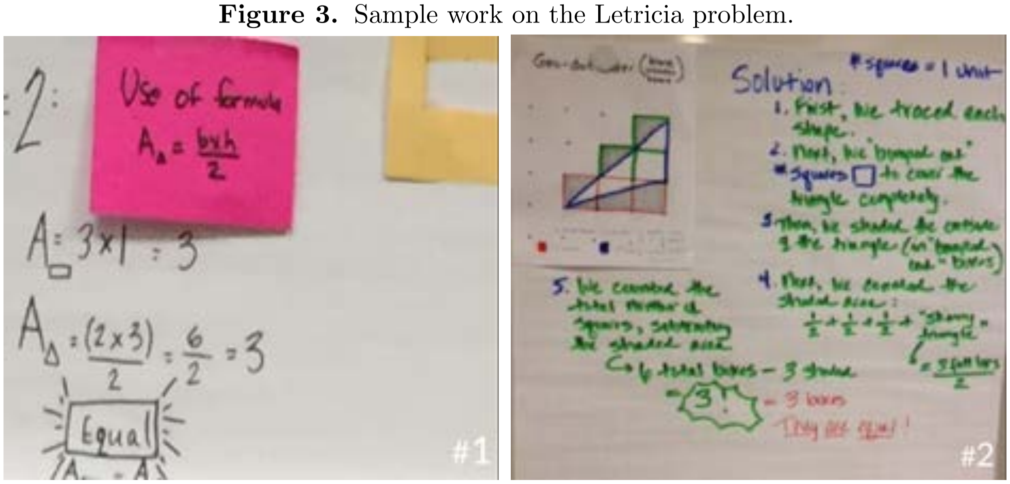
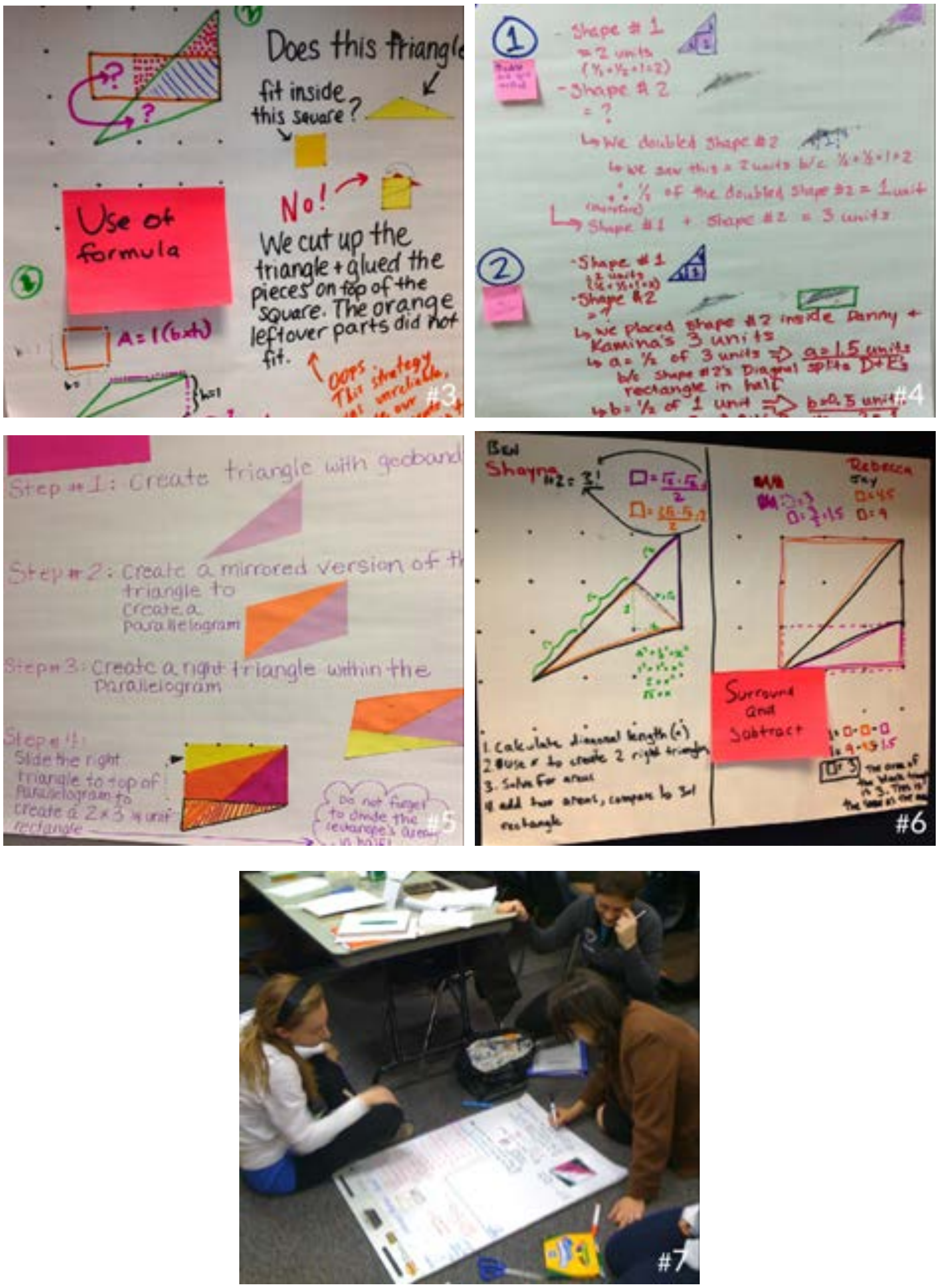


Figure 3. Sample work (continued)



Touchstone Strategy #7: Non-dominant Language

Teach one night's class in a language other than the dominant one.

One of the greatest challenges I have had in teaching at Bank Street has been dreaming up ways to help students in my classes connect social justice issues, equity issues, and discriminatory practice conversations to the teaching and learning of math. We can have spirited discussions about Geoffrey B. Saxe's seminal study of Brazilian candy sellers, which helps to break down stereotypical views of children who fail math in school. Students are amazed to see how these children failing math in school can, at the same time, run successful candy-selling businesses at the bus depots in Brazilia. It helps graduate students at Bank Street to easily see how real-life mathematical applications can motivate all learners, both in and out of school (Saxe, 1991). That being said, I still find it extremely difficult to evoke an emotional connection between this text and most of our Bank Street students' own lives. The larger percentage of our students are usually working as teachers or assistant teachers in independent schools in the New York, New Jersey, and Connecticut surrounding areas. And the bulk of our students are themselves quite privileged in that they were educated in well-resourced settings and likely neither attended schools in underserved areas nor have they yet taught in such schools.

Of course, there are always those well-versed in such issues. Often these adult learners become frustrated with the lack of awareness of their peers. They generally comment that social justice issues are too seldom naturally addressed by graduate school classmates. In my classes, I used to find they were correct. For most of the students I taught in Math for Teachers, it was often a stretch to connect emotionally with the situation that many children and teachers in inner-city schools face—where supplies are limited, funds are lacking, and quality leaders are equally scarce.

While planning for a discussion about National Council of Teachers of Mathematics' "Spotlight on the Principles: The Equity Principle through the Voices of African American Male Students" (Berry, 2004), which is additionally articulated in *Making Sense* (Hiebert, 2000), I came up with an idea. I was about to start using the geoboard for the Letricia Problem, but I wanted students to become familiar with ways first graders could creatively engage with polygons and geometry first using the geoboard manipulative. I also wanted to demonstrate a way to integrate art and geometry. I decided to teach a classic Marilyn Burns activity: the "Things That Fly" graphing lesson. In her lesson, first graders are asked to stretch a few rubber bands (called "geobands") across nail heads on the geoboards to make something that "flies." They are then to transfer their images (map them) to small pieces of geodot paper and hand them to the teacher to make a graph of "Things We Made on the GeoBoard That Can Fly." This nicely leads into graph interpretation and data analysis for young children. Each person becomes represented on this graph because there is one piece of data drawn by each child. We then discuss questions about more and less data points. After doing this activity, the kids can suggest categories of other things to make on the board (animals, vehicles, etc.), leading to various forms of classification and sorting tasks. In addition, due to the 25 points on the board, some artistic license is required and creative expression is nurtured. Nice task, I thought. Perfect for grade 1. Then it hit me. This is a task that anyone can easily follow and, though it might be a good challenge for a first grader, it is not much of a cognitive challenge for an adult. Because I wanted to marry the notion of emotion in learning math with the notion of privilege, I came up with an idea. Teach this lesson in Spanish! That semester I was co-teaching Math for Teachers with colleague Dr. Olga Romero, who was then Director of Bilingual Education at Bank Street College. We both wanted to teach by incorporating second language learning techniques and decided this task would be a perfect segue for our two skill sets to merge. We decided that Olga could teach this whole lesson in Spanish, modeling the experience of an immigrant arriving from a non-English-speaking country. As we planned the lesson, we both listed the plethora of ELL techniques that could be useful: full body expression; writing key content-obligatory words on cards and introducing each to the students with great expression; placing language peers together and interweaving the two languages during the lesson; showing pictures to attach meaning to words; repeating words over and over and affirming accurate responses boldly. Multi-gesturing would work well in this lesson. We thought about topics that might emerge for discussion after the whole lesson had ended. Most important, I thought this might bring out emotional reactions for all our students. I was psyched.

The first semester that I planned this, there were some in the class for whom Spanish was in fact their first language. I hoped that would offer them a sense of privilege. Little did we know that we would trigger a huge range of emotions in that class. I remember watching one young woman run out of the class crying and noticing clear annoyance on some faces and expressed body language showing obvious discomfort. Some had broad smiles on their faces.

Debriefing the session immediately thereafter opened the door for strong emotional commentary. I was pleased to hear from Spanish-dominant speakers, who each said, in different words, that for the first time they felt fully at home and fully "in the know." Privileged! Some non-native Spanish speakers were not so happy and expressed their discomfort. Journals after that session were filled with the full range of emotional responses. Some expressed intense anger at me for "wasting" their time. People wrote that they did not come to graduate school to "not understand" class content. This encouraged ongoing discussions regarding equity issues and issues of privilege. The "sturm und drang" that ensued ensured for me that this class session was to become one of the most important ones that I could teach. The varied emotional responses gave rise to deep and meaningful interaction.

For the past 10 years, I have always done this same lesson in Spanish and the responses each semester are strong and powerful. I carefully explain my goals for the lesson to my teaching assistants, many of whom are able to teach the lesson in a nondominant language. Dominant English speakers are offered the experience of being unable to fully understand the discourse, a feeling they may never have had. They are outsiders for the first time. Their emotional selves are challenged to tackle something they absolutely feel unable to do. They might be challenged to use all the resources around them; interacting with their table mates for help in understanding, using visual clues, depending on partners at the table who have language proficiency. Many find the work extremely difficult as they struggle to know something they don't know at first. They become sensitized to the needs of the disenfranchised. Some are enchanted with the challenge and appreciate the experience. And for the Spanish-dominant students, the glee on their faces tells it all. When teaching math, the opportunity to step into language difference is a powerful tool for feeling what "others" feel.

On feeling insecure:

> The class that Concetta spoke only Spanish to us showed me how some students feel in school who experience English as a second language. This made me feel insecure, but by the end of the class I realized there was an important message behind this. I had previously done this in my language acquisition course last semester, but this time it really connected to me. It made me realize that this is how I felt my whole life in math classes. I felt as though with more and more formulas shown to me, the less I understood and the more I felt as though the teachers were speaking an entirely different language to me. It made me understand that as an educator, it is my responsibility to make sure that not only am I teaching, but that the students are learning. There is a huge difference between the two and, often times, it is not regarded. —Erica

On learning math in a language she doesn't understand:

A guest from the second session came in to teach a lesson in Spanish. That had a pretty profound effect on me, and it's something I've been digesting since. I spoke about this a bit in class, but it really registered and I felt a range of emotions that I think are going to better me as a future educator.

The first sensation I had upon the start of the lesson was confusion—as in, what is happening? I then felt an initial twinge of, honestly, annoyance. (I took French growing up, and while my language abilities have disintegrated with time, I could communicate well enough in that scenario had the guest been speaking in French.) But anyway, that twinge of annoyance I felt was based on, "Why are they assuming every person here took Spanish growing up, or in college?" I then switched gears by turning to my classmate, Kelley, to try and figure out what was going on and also looking to others to follow along, which worked well enough for some time.

But when she started writing on the board, I distinctly remember putting down my pen thinking, "I'm out, this one is beyond me." However, when we started that class discussion after, it was a real—pardon my French, no pun intended—holy sh*t moment for me. I had completely missed the point of why we were doing that in the moment, but it struck me like a punch to the stomach afterwards. Seeing what it's like for an ESL student to learn, and how we as teachers need to reach them regardless of their English capabilities, should clearly be a top priority for anybody who might be in that scenario.

The irony is, I've TAUGHT English in Tanzania to Swahili-speaking children, which I've touched on in a journal or two before. I've also volunteered with high school students through a program called Minds Matter, which pairs high-achieving students from low-income families with a mentor, who helps guide them through the college application process and generally prepare them for life beyond. Those students who I interacted with were at that point in age all fluent in English, but many of them are first-generation English speakers and I'm pretty sure English was the second or even third language for many. All that is to say, I've been on that other side of the coin, so for me to completely miss the point of what was going on, while it was happening, was a real wake up call. But I'm almost glad that I missed it at first, because it allowed me to feel real emotions that—while only the tip of the iceberg—an ESL student would face.

I'm so grateful we had that experience because if I'm ever in that type of scenario again, I will be hyper-cognizant of exactly how I am handling myself and communicating. Not to say that I wasn't empathetic to those dealing with language barriers before, because in all fairness I was, but it was a real (sorry, gotta bring in Oprah again) "aha! moment." I've done immersive French language programs in high school where I felt completely out of my depths, but that is such a different situation. You're there in a privileged situation to learn a language, not learn math or another subject, and the expectation is that you're in the same boat as your peers. I can now only fathom how isolating it must feel for some students to feel like they're the only ones struggling to understand a language, much less understand math concepts being taught in

> said foreign language. Anyways, I wanted to get the chance to flesh out my thoughts on this because it was truly one of the most humbling, effective, and thoughtful educational experiences I've ever had, and I'm really thankful for the opportunity to have that. It'll stick with me for my entire teaching career, I know! —Caitlin

Erica and Caitlin both give us much to ponder about how we math educators can provide "original source" experience that shakes up the status quo especially for individuals who have never quite felt totally out of their league, so to speak. This touchstone strategy has worked in my classes, and I recommend it wholly.

Touchstone Strategy #8: Concept Teaching Games

Have each student plan and share their own concept teaching game.

When I started teaching Math for Teachers at Bank Street in 1974, I was committed to asking my students to design concept teaching games. During my years of teaching elementary school students and with exposure to the Madison Project math (Davis, 1984), I had used games as a medium to deeply teach the conceptual underpinnings of each big math idea. I also aimed to decenter myself from teaching so I could watch kids in process and assess their knowledge. The "concept teaching game" tool allowed me to do that. By carefully observing students at play, I could truly understand what they understood in math and what they struggled with. With that information, I could tailor plans for follow up. The games gave me the assessment information I needed, and I wanted to help other teachers learn to do that.

With that background, I decided to require the same of each of my students attending Math for Teachers. For the past 43 years, I have required the design of a concept teaching game. At the end of the process, my students' "write ups" and photos were placed in binders so other members of the class could access each game, including a description of what math concept the game taught and how to play. In later years, lesson plans for implementation were added as a requirement. All told, until the advent of Google Docs, I had about 35 years of game binders in our Bank Street Math Resource Room.

In this assignment, I always aim to carefully distinguish between concept teaching games and skill reinforcement games. They are totally different animals. For games to be classified as "concept teaching," you need to insure that the learner who has never before seen the concept, can, in fact, take ownership of the concept by playing (e.g., learn that multiplication is repeated addition or the inverse of division or can be represented using rectangular arrays, etc.). By contrast, a skill reinforcement game simply takes already learned concepts or skills and asks kids to practice them as known facts (e.g., 8×5 does equal 40). A skill reinforcement or practice game might ask the player to pick a card like 8×5, answer it, and move one space on a board. Nothing in the game invites the learner to see why $8 \times 5 = 40$. They do not see an array with grid paper; they do not have to collect 5 towers of 8 cubes or 8 towers of 5 cubes, or even use a hundreds board to skip count by 5s or by 8s the required amount to get to 40. Such a skill reinforcement game does nothing to insure conceptual understanding of what 8×5 looks like or the multiple ways you can deconstruct 8×5 [as $(5 \times 5) + (3 \times 5)$, for example].

Leah Silver, my teaching assistant on and off for the past two years, developed additional tools to help our students appreciate the continuum of concrete to abstract thinking required by concept teaching games. Students were asked to design games that focused in the concrete/pictorial realms with only advanced levels moving to the abstract realm.

I have come to expect that people would resist and balk at being asked to make their own concept teaching game. And a few did every semester. Even some faculty told me they felt the concept teaching game assignment was burdensome, anxiety provoking, and not necessary, given all the fine new curriculum materials people now are using in math classes in elementary schools (e.g., Investigations in Number Data and Space, Bridges, Math in Focus, Envisions, etc.). I resisted their resistance because of what I was reading in my students' journals each and every semester.

Students often struggled to decide what a concept was, struggled to select one, were challenged to think about how they could teach something brand new to children, and struggled with design elements and how to engage children. I say, that is good! Their job as new teachers is to learn all that.

The frustration these adult students share with me and my assistants give us additional insights into who they are as teachers, what math knowledge they bring to this problem, how well they take our advice, how skillfully they adapt our advice, and how true they are to the purpose and constraints of the assignment. The emotional pleas and draft ideas they present to us in journals provide me the missing link for me as their teacher. As a course instructor, I rarely if ever see my students teaching. But I can read their thinking about "teaching" when they write me with nascent ideas regarding picking a concept (not just a skill), planning how to teach a new concept, planning how to keep kids engaged, and planning how to use the idea to assess learning. This is as close as I can get as a course instructor to seeing what goes on inside a teacher's mind when developing plans to enact teaching.

Every week I would model a well-crafted concept teaching game and its rules so they could taste a wide range of them all semester. Some that we always used were: Action Fractions, The Joining Board Game, How Big, Make Ten, Build a 100, Racing Dice, Capture Five, The Factor Game, Target 1000, and Make Five. Slowly they become more and more aware of what a concept teaching game could look like.

Because I want always to model teachers working collaboratively as a community of learners, only six people per session are asked to share their progress in designing a game. The other 20 or so teachers were free to roam around the room playing each game and could offer suggestions for clarification and perhaps improvements. No teacher should ever feel alone in developing teaching methods or ideas.

My underlying goal in this assignment is to assess if an adult student is capable of designing a complete math experience that focuses on a learner-centered activity and gives ownership of the math to the student. I know this is a challenge, but why is a teacher attending Math for Teachers if not to learn to do that? As you will see from the two journal comments below, emblematic of many entries I have read, the concept teaching game assignment is challenging at first. I believe this challenge, once met, is usually felt to be very worthwhile and I recommend this activity be implemented in courses for new teachers. Once again, I learned about the students' tenacity for sticking with an idea for a semester and revising and improving it based on feedback from both the instructors and fellow students. Though other faculty might not agree with me, I feel this was one of the best assignments I could have given my students. I learned so much about them and I had fun with their creativity as well. I cannot tell you how many of our students' games came with me to districts across the US and to teachers in other countries as well.

Figure 4. Sharing a concept teaching game

On making a game:

> I actually love the game-making experience much more than I thought I would. I had a mental impediment with it. I think part of it I can attribute to the fact that I just don't have experience creating lesson plans or games, and it was all sort of new and uncharted territory. I had a couple false starts with creating the game—going down one direction and then abandoning it altogether. But it was such a helpful and clarifying experience. My classmates—and Hal!—had such great ideas and constructive criticism about the various directions my game can ultimately go in. I thought that I would treat this game as purely a class assignment, and not necessarily something I would take with me, but I couldn't have been more wrong. I'm going to continue tinkering with this game through the years, and ultimately I would like to develop this into a "chutes and ladders" style game, like Hal suggested. How I'll execute that is a bit of a work in progress, but I'm thinking on it and am genuinely excited to have a polished product to take with me into classrooms. And I can even do different iterations of the game. But it was a huge mental hurdle that I got over, and I'm genuinely excited to create more games in the future. —Caitlin

On overcoming fears and meeting goals:

> To address the very first point, about taking risks: I think I've progressed in this area. The piece of this course that helped me most with this was the concept teaching game. It was the assignment I was most afraid of—spearheading a whole game all by myself and then teaching it all by myself to classmates that I've seen excel all semester. I was terrified. Fortunately for me, the scheduling put me at the very beginning which meant I needed to jump in with both feet. I asked a classmate if she might play the game with me beforehand and she agreed! That was a risk because I let someone give me feedback, though it was also a safety net because it helped me preview the work

> in my preferred modality (one-on-one). This reminded me how some of my students may need the chance to preview a project in a different (possibly smaller) setting before presenting it to the class as a whole.
>
> In presenting it to small groups, I was impressed with the feedback I received. I was also surprised how well I was able to hear my classmates. I thought I was going to show the game, explain it, and then everyone would tell me that I was unprepared and the game was stupid. This did not happen. I got some feedback that helped me develop my core idea and also inspired me to change a few simple elements to make the game more accessible. I was able to move my Ego out of the way and listen to the feedback as it was coming to me.
>
> I found this experience to be helpful to both of my goals: I was able to successfully take a risk and I also learned some ways to support different kinds of learners. I noticed that I needed some extra one-on-one time, which could be something I could pass on to math students in the future. I also noticed that my classmates had some trouble with the organization of the game, which showed me clearer ways to display information for all types of learners. Yay! —Holland

As each semester came to an end, I grew more and more excited to see what the final four sessions would bring. For the 30 to 40 minutes of each two-hour session, students had signed up to share their games at each of six tables in the room (no more than six games each week). The balance of the class would rotate and play each game for just a few minutes. It was like going to a game fair for the last four weeks of the semester. Everyone would get a sense of the purpose of the game (the concept it teaches . . . or fails to teach) and how to play. Then they were asked to give hot and cold responses to the game maker. Here again I put each teacher into the position of being a professional supporter of a colleague, sort of a critical friend who can give substantive teaching support to one another. Another goal was to create and model what a supportive educational community looks and feels like. I aimed to provide emotional support for the community of learners. As my assistant and I roamed the room, we not only saw the evolved games being played, but we also watched and listened to the tone and content of the support that other students are able (or not able) to provide. Once again, the emotional tone in the room is what I am learning from. That gives me the opportunity to observe, follow up with each teacher, and provide thoughtful productive commentary on each person's teaching, conceptual understanding, and capacity as a constructive colleague.

See samples of concept teaching games that were shared in Figure 5 (next page).

Figure 5. Sample concept teaching games

Conclusion

Bringing joy to uninspired teachers of math.

My dream has been for a generational shift in math teaching spearheaded by the teachers who graduated from the Teaching and Learning Department at Bank Street. In my work, I have intentionally linked a teacher's emotional life to their thought process during a math teaching re-education process. One's feelings or affect and one's intellectual engagement to study are always intertwined (Brakett, 2017). In mathematics, I have found that negative feelings about the subject often overpower the intellect. If the feelings towards math are negative and are not challenged and remediated, the potential effect on our graduates' classes of children will remain bleak.

Every one of our adult students had already been through grades pre-K - 6 math prior to attending our class. So the math we teach in Math for Teachers is not new. It may, however, be delivered with a very different focus. I advocate both an inspired learning process as well as an unlearning/re-learning process. To inspire other teacher educators, I hope I have built a case for addressing emotional impediments and negative triggers in math learning and the reasons to provide positive replacements for those feelings for teachers. Teachers for years have told me that their perceptions of themselves as mathematical thinkers change as a result of attendance in Math for Teachers (Melnick, 1992).

In recent classes, due to my longevity at the College, I have had teachers announce on day one that their mothers took classes with me in the 1980s and, to this day, recall how powerful their relearning of math was. The residual learnings from Math for Teachers can be outstanding. I feel it is largely emotional. I also believe that most teacher educators in the country can do this if they take care to value the emotional learning experience equally with the cognitive work. Learning math in the way we have come to teach it at Bank Street can change the lives of future adults in our society. If we sensitively plan for change and remain excited about the prospect, change will happen.

A graduate school teacher educator usually can only assess what his students learn by observing them in the graduate classroom or by requesting papers written during the instructional semester. Course instructors rarely have the time or the luxury of visiting each student in their field placement. I know I often wonder what has been learned in the long run. Did anything that I taught stick? Sometimes we find out.

In the last paragraphs of this paper I share with you a description by a student who attended my class seven years ago in 2011. Tyler Jennings took one course with me to test out Bank Street as a potential place to do his graduate studies. He was, at the time, an experienced fourth grade teacher at Caedmon School on the Upper East Side of Manhattan. While at a social gathering this past month, I happened to overhear Tyler describing to others how, from the time he was a child, he had always disliked mathematics until attending my Math For Teachers course at Bank Street. He described that experience as transformative and said it totally changed not only his relationship with mathematics, but also his career trajectory. I close with his (and one other) story because both capture nicely how teacher education must address the emotional as well as the cognitive if it is to be effective. You need to teach "affectively" to be effective.

Tyler explains what he calls "Tyler's Math Journey" from a math phobe to a math coach, working as a professional developer with teachers today at his school on the Upper East Side of

Manhattan. Listen for the emotional, address the intellectual, help teachers connect the two. Help them appreciate mathematics for what it always has been —simply the study of relationships and the science of pattern. Help them to feel it and then leave them to inspire others.

Here is Tyler's story:

> Until the age of 25, I believed that I was not made for math. As far back as I can remember, certainly third grade, I knew with conviction that I was a poor math student and that I disliked the subject. I noticed the way that adults spoke differently about me when the subject of my math learning arose; the tone in their voices changed, became pinched and awkward rather than proud. Sometimes I would imagine an expert who could enlighten me: "It's really quite simple," they would say, "you don't have a math mind, but you can be good at other things." I ceased to pursue my math studies as soon as possible. If a high school math course was not mandatory, I did not take it. I chose a university without core requirements, so that I could avoid college math altogether.
>
> Today, I work as a Math Coach at a pre-K and elementary Montessori School in New York City. Since I also serve as the school's Director of Curriculum and Innovation, I work with teachers in other subject areas, too, but I know that I behave differently when I approach math curriculum and instruction: I come alive. I feel that I am at my most passionate and engaged, successful and talented, helpful to teachers and impactful in children's lives. I conduct far more professional research into math content than any other subject—I can't get enough of it!
>
> So what happened? At the age of 25, when I enrolled at the Bank Street College of Education, and specifically in a course called Math for Teachers, a dangerous cycle was disrupted. I was invited: 1) to share my math journey, including its emotional dimensions, and 2) to experience math differently. I was welcomed in; it was made immediately clear that, at Bank Street, there would be no outsiders to the pursuit of math learning—that math learning could be for all, could be for me. And we did math together, which made it possible to redefine our very concept of what math is: an active pursuit of meaning-making, the study of the science of pattern. So much more than an inert body of knowledge to store (or fail to store) in the mind, which is, of course, the way that I was taught math. I cannot express how cathartic, emotional, and exciting that process was, and continues to be even as I write this.
>
> Without my transformative experiences in this teacher education program, I know that my own negative experiences as a young math student would have directly informed my choices as an educator. Not only would I have mimicked the way that I was taught, but I would have communicated, subliminally or not, my dislike of math to the students. I could have easily become the expert who told certain children that they simply did not have a math mind, but that they might be good at something else. It seems to me that an excellent teacher education program may be the best chance for disrupting this dangerous cycle for future educators. It certainly did for me. —Tyler

I wholeheartedly believe that the impetus for the transformation Tyler speaks of is Bank Street's longstanding commitment to balancing theory with practice. Instruction in all our math classes consciously aims to balance current, well researched theory equally with the emotional experience each teacher brings to her studies. The catalyst that connects theory to practice, public knowledge building to personal, is deep reflection.

Let's hear from Caitlin one more time as she reflects on the course one full year after its completion. What is it that triggered her massive change? What was it that truly changed?

> I really remember benefitting from you asking us to dig deep in our own personal math experiences. My math experiences were happily filed under "The Past: Do Not Open." And, for lack of a better comparison much like therapy, it's by revisiting and delving into it that we are able to grow, heal, and move on. You brought joy back into the subject with in-class projects. I got so into the making my own concept teaching game in particular, it really returned ownership of the subject after all those years. —Caitlin, Spring 2017

This year, I have begun to visit local schools to meet with and observe teachers who attended Math for Teachers at Bank Street, and whom I have taught. In the New York City area, I have seen the next generation of math teachers from Bank Street flourishing. Math in these schools is taught with feeling by inspired teachers and with math-inspired children. In more than one of those schools, math has become the students' favorite subject and, in some cases, the teacher's favorite subject to teach.

What does that foretell? Math teaching can change for all people if we ensure that teachers see math in all its dimensions and recognize both the emotional as well as the intellectual.

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We're All Human, So Why Does Equity Matter?

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Abstract. There is a need for mathematics instructors in higher education to have more equitable teaching practices. One way to address this need is through equity-centered professional development (PD). Using interviews across one year of professional development (PD), we describe how three immigrant mathematics instructors grapple with incorporating equity in their teaching practice using Gutiérrez (2009) four dimensions of equity. We found that the instructors focused primarily on awareness and access with respect to equity but did not understand how to infuse equity in their practice. We also found that three themes, the discipline of mathematics, all students are human, and the identities of the instructors and beliefs seem to serve as barriers for instructors' implementation of equitable practices. Future research might consider how PD could be informed to account for the themes found in this study.

"It doesn't matter what my race is. Um, or my gender, because, you know, I'm teaching a subject that I feel is just it's universal. So it really does not matter." This quote came from a two-year college mathematics teacher. The statement is not surprising, and, in some cases might even be expected, as mathematics as a discipline has historically been positioned as universal, unbiased, and politically neutral. The implication is that mathematics is accessible to everyone. While this perspective is common, it is problematic as it removes any accountability or responsibility from the role that mathematics as a discipline plays in what it means to do mathematics and who can do mathematics. This perspective is transmitted to individuals who attain advanced degrees in mathematics and to the students they teach. Having this perception makes it difficult to understand why equity is important in the teaching and learning of mathematics.

In this paper we focus on three two-year college mathematics instructors who are immigrants. We sought to understand how they negotiate their perspective of mathematics, which is closely aligned with the opening quote, when trying to understand and incorporate equity in their classrooms. The research question that guides our paper is: *How do instructors think about their teaching practices while grappling with tensions around self-identities, as well as personal experiences with and beliefs about mathematics as a discipline?*

Literature Review

Even as efforts are being made to advance STEM fields toward a more humanistic and equitable endeavor, these fields tend to resist transformation (McGee, 2020). Given that mathematics is foundational in STEM, one possible explanation for the slow change within these fields can be traced back to how mathematics is perceived in society. Leyva and colleagues (2021) argue that mastery of mathematics is often seen as a precursor to one's social standing and frequently used as a measure for intelligence (Hatt, 2012). Similarly, Moses and Cobb (2002) liken math literacy to civil rights, emphasizing its societal importance. Hence, when discussing mathematics as a discipline, we need to recognize the high esteem in which it is held in society and how it functions as a form of cultural capital (Williams et al., 2016).

In the pursuit of equity in mathematics education, it is crucial to examine four narratives of mathematics as a field of study: the perceived universality of mathematics, the understanding of what it means to engage in mathematical practices, the historical exclusivity of the discipline, and its inherently political nature, especially as these factors relate to the often-understudied in the context of community colleges. First, mathematics is often conceptualized as universal. That is, mathematics is often portrayed as a universal discipline, suggesting its omnipresence and accessibility to all, irrespective of race, gender, or any other identity markers (Baber, 2015; Battey et al., 2023). The second narrative to consider when thinking about equity in math education is understanding what it means to do mathematics. Understanding what it means to do mathematics is critical as there is often a misalignment between what it means to do mathematics and how mathematics is taught (Brown et al., 1989). The contrast between the two – school mathematics and the authentic practice of mathematics outside of the classroom – highlights the tension that exists between what it means to do mathematics and how it is taught in the classroom (Boaler, 2000; Williams et al., 2016). The third narrative is who is permitted to do mathematics. Historically the discipline of mathematics has been exclusive, highlighting that only certain individuals can do or are good at mathematics (Ernest et al., 1989; Hottinger, 2016; Leyva et al., 2021; Martin, 2009; Nasir et al., 2011). The fourth narrative is recognizing that mathematics is political (Gutiérrez, 2013; Stringer et al., 2022). The classroom is more than just a site for social reproduction and enculturation. Rather, it is a space where power dynamics, identity, and cultural constructs intersect. Mathematics inherently carries political aspects and power dynamics similar to other human activities. Collectively we argue these four narratives about mathematics inform instructors' teaching practices and how they conceptualize equity. Additionally, we build upon the literature by focusing on a population and context that is often understudied in mathematics education, immigrant instructors in mathematics and community colleges.

Theoretical Framing

The four dimensions of equity is a framework as described in Gutiérrez (2009) articulates considerations for equity in mathematics education. The framework highlights four components, access, achievement, identity, and power. The components are divided into two axes—the dominant axis and the critical axis. The dominant axis represents the prevalent emphasis for equity in mathematics education which centers on access and achievement in which access precedes achievement. Access refers to ways in which students can participate in mathematics. This includes students engaging with teachers, curriculum, and resources. Indeed, access may impact achievement. Achievement attends to student outcomes such as grades and scores on standardized tests and can highlight ways in which students may demonstrate their mathematical knowledge. While the dominant axis is important, Gutiérrez acknowledges that it is only a partial view of equity in mathematics education. The second axis which is referred to as the critical axis, includes identity and power, where identity precedes power. Identity refers to the social markers of students and teachers, as well as their lived experiences, and cultural socialization. Understanding and incorporating students' identities is important to attend to equity. Power, the last component, highlights the power and agency that both students and teachers have and how teachers can use their power to ensure that classroom instruction is inclusive and attends to the identities of students.

Methods

The Mathematics Persistence through Inquiry (MPIE) project is a five-year NSF-funded project whose goals are to 1) study a two-year college's response to a state-mandated change in gateway mathematics courses (College Algebra, Precalculus, and Trigonometry), 2) use cycles of design research to build the capacity of math instructors in the two-year college to foster student success, and 3) investigate the effects of the capacity-building effort. The use of design-based research has enabled the building, implementation, and subsequent refinement of a professional development (PD) program for two-year college mathematics instructors. The focus of the PD program is on supporting the development of inquiry- and equity-focused teaching practices. The PD program for the research participants discussed in this manuscript took place during the fall and spring semesters. The fall PD used Smith and Stein's 5 Practices for Orchestrating Productive Mathematics Discussions (2011) as a template to support teachers' practices around inquiry and equity. Additionally, PD participants had opportunities to engage in mathematical tasks, apply such task frameworks to their own teaching, and reflect on their practice. During the spring instructors met five times for one hour each. The spring PD involved the use of Equity QUantified In Participation (EQUIP) tool created by Dr. Daniel Reinholz and Dr. Niral Shah. EQUIP is a classroom observation tool designed to "illuminate patterns in student participation" (Reinholz and Shah, 2018, p. 141). EQUIP was used as a support for teachers to better understand teaching practices and reflect on how to make changes that made their teaching more inclusive and equitable. Members of the project team analyzed videos of classroom instruction and created reports that were shared with participants. The regular sharing of personal teaching reflections and experiences with each other fostered the development of a small professional community among participants, which allowed for the sharing of challenges and ideas around teaching.

Engaging in equity- and inquiry-focused PD over the course of one year allowed for participants to develop an understanding of inquiry and equity and its application to their teaching, engage in continuous refinement in their teaching, and make connections from their learned practices to their local context. In this paper we discuss participants' evolving perspectives on equity, which are connected to their own personal experiences and relationships with mathematics, and, in turn, impact their teaching practices.

Setting and Context. The setting of this research project and PD efforts is a two-year Hispanic Serving Institution (HSI) in the Southwestern United States. In this paper we will refer to this institution as Southwestern HSI (SHSI). SHSI serves a student population that includes a majority of students from historically minoritized communities including Latinx students (68%) and students from low-income households (70%). PD participants include SHSI instructors who primarily teach gateway mathematics courses.

The MPIE PD program is a two-semester program running through the fall and spring terms. The participants discussed in this paper participated in the MPIE PD during the fall 2021 and spring 2022 semesters. All were math instructors at the SHSI, most of whom taught a variety of courses including gateway mathematics courses. The fall semester PD focused on inquiry-based teaching and learning with participants meeting six times, two hours each (for a total of 12 hours). In the spring, the PD shifted to an equity and inclusive emphasis, with participants meeting 4 times, 1 hour each (for a total of 4 hours). The reduced PD session time in the spring allowed for individual classroom observation and debrief sessions.

Data. For this paper we present three case studies from the same cohort of teachers: Paul, Nhung, and Savana. We selected them as cases because they are all immigrants with unique experiences that seemed to have shaped their beliefs about and ultimately their teaching of mathematics. The beliefs they shared highlighted a tension between their perspectives on what it means to do mathematics, what mathematics affords as a discipline, and the role of equity in disrupting their perspectives. Below is a brief description of each participant.

- Paul is a full-time mathematics instructor who grew up in Tijuana, Mexico and identifies as a Hispanic or LatinX. However, he has been perceived by others as white in some spaces and Latino in others, which has created a complex, dual identity for himself. He has taught a variety of courses at SHSI for more than two decades.
- Nhung is a full-time mathematics instructor who identifies as Vietnamese with Chinese Ancestry. He spent the first 11 years of his life in Vietnam before coming to the United States as a refugee. He has taught a variety of courses at SHSI for more than a decade.
- Savana is a part-time mathematics instructor who identifies as Hispanic. She grew up in a large family in Tijuana, Mexico and is a first generation college graduate. She has taught a variety of courses at SHSI for more than two decades, as well as at other local colleges.

Table 1. PD Participant Demographics Information and Employment Status at SCHSI

Participant	**Gender**	**Self-Identified Race/Ethnicity**	**Status**
Paul Martinez	Male	Hispanic or LatinX	Full-time
Nhung Tran	Male	Vietnamese with Chines Ancestry	Full-time
Savana Sanchez	Female	Hispanic	Part-time

The data examined from the participants are three, one-hour interviews each across two semesters of PD. The first interview occurred toward the end of the fall PD, the second interview occurred before the start of the spring PD, and the third interview occurred after the conclusion of spring PD. Each interview was semi-structured and designed to capture information around the instructor's teaching background and style, mathematics experiences, equity beliefs and practice, and inquiry beliefs and practices.

Analysis. Each set of interviews were analyzed by two different researchers. The first round of coding involved members of the research team writing memos so that common themes regarding equity and identity could be identified, with Gutiérrez's four dimensions of equity as a guiding framework. After discussion, the themes of *self-identity*, *math as unbiased/universal*, *we are all human*, *awareness*, and *access* were identified as themes that appeared in each participant's data. Below we unpack how these themes address the research question: *How do instructors think about their teaching practices while grappling with tensions around self-identities, as well as personal experiences with and beliefs about mathematics as a discipline?*

Findings

All themes were present in each set of interviews. In the subsequent sections we highlight how participants' personal experiences and perspectives show up in their teaching practices. For example, instructors found ways to connect to their students through shared lived experiences.

Educational Experiences and Identities Impacting Practice. All three participants identified as immigrant teachers - each grew up in another country and then became a teacher in the United States. Their unique experiences growing up and becoming immigrant teachers haves played a role in their teaching, impacting their practice and the way in which they interact with their students. Below we briefly unpack each participant's educational experiences individually - highlighting significant stories as they relate to their identities in the context of teaching and learning.

Paul grew up in Mexico and identifies as biracial. In Mexico he experienced racism and marginalization. Often referred to as "Gringo" or "Blanco", he struggled to fit in or feel like he belonged in his high school classes. As a 15-year-old, he took a difficult physics class which he ended up failing. During a class period towards the end of the term his Physics teacher stated in front of the class that "The American white boy failed physics. So now he's going to learn what it is to be oppressed and what it is to be privileged." This experience was "hurtful" to Paul and has impacted his choices and values as a teacher. As a teacher at an HSI on the border with Mexico, he knows that there will be students in his class who will likely have an identity or experiences like his own. Having such students in his class is an opportunity for him to connect and provide the support students might need. He is fluent in Spanish and will share his identity with his students to let them know that he also has experienced challenges in his education.

Nhung spent his childhood in Vietnam. He was 11 years old when his family moved to the United States (US). Growing up speaking Cantonese, he knew very little English at the time. Thus, as a student in US schools, he struggled in his classes and had multiple uncomfortable experiences when being called on during class. The language barrier made it hard to "verbalize" his thinking and these negative experiences impacted his confidence and learning processes. Mathematics classes usually did not pose the same level of difficulty for him, given the reduced amount of English vocabulary. However, he did struggle to read and interpret contexts presented in mathematical word problems, which made it difficult to apply the appropriate mathematics. These past experiences impact how Nhung teaches. For one, he never wants to put students in a position where they feel singled out and uncomfortable, and Nhung mentions "cold calling" as a strategy he disliked as a student and does not do. And second, he wants to be available to help students if they are struggling and need additional support. He goes further to say "I feel like I can relate to students better because...the adjusting to classrooms in the United States is just something that I have a little bit more appreciation maybe for what they have to go through." However, this responsibility lies with the student, as Nhung does not typically engage with a student unless they take initiative (e.g., vocalize specific questions or demonstrate engagement with the content).

Savana was born and raised in Mexico. She was the youngest of 13 children and grew up in a traditional household. Neither of her parents were formally educated, so learning and writing were unfamiliar to them. Her father believed that the priority for women was to focus on family, children, and the home, but her mother viewed education as important despite not having the opportunity to attend school herself. Savana embraced both values in her own life - family and education. Thus, Savana attended school through community college in her hometown of Tijuana, and then moved to the US to pursue her dream of becoming a teacher. This was not easy. She did not speak English, but she was motivated. She had a "dream...[to] teach math", and even though she did not know the language, noting that it "was really, really challenging", she strived to follow her dream. This aspect of Savana's identity is a big part of who she is and impacts how she interacts with students. She wants students to know and understand that pursuing an education (especially when not in your native language) is "not easy, but it can be done. If we work hard." Savana shares this story with her students, hoping it will motivate them to continue pursuing their education.

These instructors understood and could relate to the challenges that learning a new language or educational system posed to students in their courses. Instructors shared these challenges with students and saw themselves as potential role models (e.g., Savana). Further, these instructors internalized experiences in a way that influenced how they interacted with students, empowering them to employ teaching practices that countered what they experienced as learners. In Paul's case this meant trying to provide additional support. However, in Nhung's case he often centered his own negative experiences, which reinforces inequities in the classroom by unintentionally excluding students from participating during classroom instruction.

Mathematics is Universal and We're All Human. An interesting theme that arose across the interviews was the idea that anyone can do mathematics and that mathematics is a neutral subject. In fact, Nhung felt that mathematics is "unbiased", and that success is achieved with enough "time and effort". He went further to note that that was one of the motivating reasons for joining a PD that had a focus on equity - he wanted to understand how such a neutral subject could have inequities.

The notion that mathematics is a neutral subject was also intertwined with the idea that "we are all human". When asked about how she thought about race, Savana stated that she did not think about race and went further to say: "We are all the same. We're all human beings. We're all the same." While Paul had a similar sentiment, he situated it in the classroom setting about engaging with mathematics and promoting student success. Paul noted the sameness in which he wants to support students while still monitoring biases he may have:

> I'm trying to break the stereotype by, by trying to be okay now we can... be the same with everybody. Be the example, communicate the same way with everybody, treat everybody with the same level of respect, communicate with the same level of acknowledgement of respect. Don't focus on only a handful of students or one gender. Everybody's the same, everybody's equal. We all belong to one to race the human race. If I, if I make sure that everybody to me is a human, that right there is helping me keep myself in check and avoid falling into those biases, those stereotypes. And if they see that I'm doing that and, and it's encouraging them to succeed.

Throughout PD, Nhung struggled with the idea of inequity existing in the mathematics classroom:

> I just have always seen myself as I'm just a math teacher. Right. It doesn't matter what my race is. Um, or my gender, because, you know, I'm teaching a subject that I feel is just it's universal. So it really does not matter.

Although all participants had this idea of mathematics being universal or we are all human, they each acknowledged the inequities that they themselves had experienced as students or inequities they recognized as being present in mathematics as a discipline. These inequities shaped their identity as mathematics teachers, and, in turn, impacted how they interact with students.

Getting Stuck: Awareness and Access. Throughout the interviews it was clear that Paul, Nhung, and Savana increased their understanding and awareness of classroom equity issues as they progressed through the PD. Participants seemed to think about equity as access to opportunities. An instance that captures such awareness that was developed early on is when participants were provided with an image, which is now ubiquitous, showing two cartoon panels. In one panel, there are three children trying to look over a fence to see a baseball game. The children are of different heights, and each child is standing on the same type of box to try to look over the fence. Although each has a box to stand on, not all can see over the fence. In a second panel, each child is standing on a different size box that will allow for them each to see over the fence. Although this image does not capture equity as a concept in its entirety, it provides an entry point to a conversation about equity, especially for those who are new to having such discussions. It was this image that gave Paul clarity about equity in the classroom for the first time:

> And they showed the [cartoon description] and I thought to myself, by the way, please forgive me if I if I release a colorful metaphor, because I really get worked up about this. But I was thinking, "God damn w– we've been doing this all along! We've just been giving everybody the same box!" But hey look– there's tutoring, whoopie! Hey look– we have workshops, yee haw! But if students have different needs, how can we bring them up to the same level instead of just giving everybody the exact same opportunity? That's when it clicked, that's how it's done.

Paul realized that giving all students the same opportunities was not what it meant to attend to equity. He worked to meet the students where they are, and part of this process was allowing students to participate in ways that they feel comfortable. This notion of opportunity and comfort was also reflected in Nhung's and Savana's interviews.

Nhung felt that being equitable was giving "students the opportunity to learn" through group activities guided by "leading questions". He believed that each of his students were capable of the same achievement with enough practice, time, and effort. However, Nhung began to struggle with his definition of equitable teaching once he began to realize the varying levels of preparedness in his students, which impacted their ability to take advantage of such opportunities. He noted:

> Well, when you interact with students when you recognize that they are not as prepared to take advantage of those opportunities. Right? And now I see where some of it is falling short. But I don't really know how to address it yet. . . Right. But it's not just equal opportunity. Right. Because if you start further up than someone else, and both of you are presented with the . . . same opportunities, you are going to rise or at least you have the opportunity to rise, um, at a greater rate than someone else who started way behind.

While Nhung felt opportunities were important, he recognized that giving students the same opportunities did not address the gap that existed between students. This tension caused Nhung to shift his perception of equitable teaching slightly, adding the caveat that students who entered his class with gaps in prerequisite knowledge just needed more chances to work one-on-one with him as the teacher.

Nhung also shared that if students "elect to not interact with" him, he will leave them alone and not "bother them". From Nhung's perspective one-on-one attention was a way to elevate students to a level commensurate with their classroom peers. Savana had similar sentiments stating that equity "means being able to provide every student what they need", which may be unique to each student. Ultimately Nhung viewed issues related to equity as access issues and students just needed opportunities to engage with mathematics. Savana, like Nhung, recognized that students were at different ability levels and felt that support needed to be customized based on a student's specific needs. However, while Nhung struggled to attend to students' needs because one-on-one help during class was difficult to accomplish for all students, Savana expressed not knowing how to begin to attend the issue:

> How do I help the student that, um, you know, uh, that has, I don't know, anxiety and, and, and, and just being asked to, to talk in front of the class, you know, makes just asking that person, a question, you know, uh, may block him and, and not be able to, you know, to, to, to follow, you know, what I'm doing. So it's, so it's so, uh, challenging.

In the quote above we see that Savana recognized potential barriers for students' participation and acknowledges potential consequences for different moves she might make, however this leads to paralysis in that she gets stuck in her awareness and does not try moves that might be more equitable.

By the end of the first year of PD not much had changed with perceptions of equity in the classroom. Paul, Nhung, and Savana all discussed increased awareness about equity in their teaching, but little change could be seen in the data across their interviews. All continued to see increasing access to engaging through participation in mathematics as the key piece to attending to equity in the mathematics classroom.

Discussion

Paul, Nhung, and Savana provide important insight for what it means for community college mathematics instructors to grapple with incorporating equity in their teaching practices. Although each instructor had increased awareness with regard to what it meant to attend to equity in the classroom, at the end of their first year in PD that awareness was limited to focusing on increasing access to mathematics. This translated in their practice through their teaching by having more one-on-one interactions with students, increasing availability to students, and encouraging more questions and discussion from students (as students felt comfortable). However, in some cases, this puts the responsibility upon the student to take advantage of the additional access, which assumes that more access is what students need and that the students can accommodate more instructor access into their lives. This perspective presents a somewhat limited view of what it means to attend to equity.

There are several factors that might contribute to this limited view of equity in the classroom. Through themes identified across Paul, Nhung, and Savana, potential barriers to instructional change seem to be their own personal identities, beliefs, educational experiences, and beliefs about how to be successful in mathematics. For example Nhung recalled how his own experiences might explain his expectations of English Language Learners or why he does not "cold call" students. In addition, the perspective that mathematics is unbiased or universal makes it difficult for the instructors to understand how shifting teaching practices beyond more "opportunities" would help attend to equity. This idea is further validated as Paul, Nhung, and Savana succeeded academically and professionally, in part because they took advantage of opportunities and were motivated. For example Savana, who is a Latina woman from Tijuana, described her upbringing as one where women stayed home and did not further their education, so her decision to pursue a math degree and learn how to speak English provided her with the perspective of "If I can do it, you can too". Each participant had this view to various degrees and as a result, this "pulling-oneself-up-by-the-bootstraps" mentality makes it difficult for these instructors to shift their practice beyond creating opportunities and encouraging participation.

Conclusion

Moving forward we are working with instructors as they grapple with challenges related to inquiry and equity in their classrooms. For example, Nhung has identified ways in which he can encourage student participation that is not "cold-calling". Specifically he is speaking with students individually first before asking them to present their ideas. Getting students involved in this way helps position them as having more ownership in their engagement in the classroom. Similarly, Paul has put forth more effort to better understand and attend to his students' needs. For example, he recognizes that helping students is not just about providing extra office hours but also understanding that students can benefit from flexibility in a course (e.g., due dates). Minimal shifts in course constraints on Paul's end (e.g., homework due dates) can make a big difference for students who have limited resources or unique circumstances (e.g., limited access to wifi). The realization that Paul had this power occurred to him during the PD. Lastly, Savana is using small group work as a way to support students that might need the extra support. Although she recognizes that small group work might come at the expense of covering content, Savana sees it as an opportunity for students to support one another. Further, this allows for her to be more strategic in supporting students, especially those who have greater need, and ultimately meeting students where they are.

In this paper, instructors highlight a tension that exists between their own identities, beliefs, and experiences, and the common narratives that exist within mathematics (e.g., mathematics is universal). Understanding these tensions and the deep-rooted impact of experiences with and beliefs about mathematics have given us a better understanding of how to reframe aspects of ongoing PD to try to empower instructors to move beyond awareness and access and towards practice that also incorporates Gutiérrez's critical axis dimensions – identity and power.

Acknowledgments

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Titan Mathematics Apprentice Program (TMAP)

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Abstract. The Titan Mathematics Apprentice Program (TMAP) was created to support the commitment at California State University, Fullerton (CSUF) to providing students with work-related experiences. Dr. Ichinose, who also coordinates the Single Subject Mathematics Credential at CSUF, designed TMAP so students can gain experience in teaching mathematics through lesson planning, classroom delivery, and mentorship. The program gradually increases apprentices' responsibilities, helping them develop essential teaching skills such as classroom management and student assessment. TMAP also fosters professional interactions and encourages reflection on teaching practices, building confidence and practical skills for future mathematics educators. Over 200 students have participated in TMAP since it began in 2017, with the 2024-2025 academic year consisting of 46 apprentices. The program follows a traditional apprentice model, incorporating elements of supplemental instruction and embedded tutoring to enhance learning by doing, mentorship, and gradual skill development.

Introduction

The idea of an apprentice or apprentice program is not new. For years the term "apprentice" included craft/trade apprenticeships, medical school students, technical apprenticeships, and graduate apprenticeships (Hawkins, 2008). The Titan Mathematics Apprentice Program (TMAP) follows similar approaches with the major focus on learning by doing, mentorship, and gradual skill development. These have been noted as essential characteristics of any effective apprenticeship, including those in trade, medical and educational settings (Billett, 2010; Fuller & Unwin, 2003; Raelin, 2008).

TMAP Builds on Learning Assistant Models

Apprenticing in the TMAP program at California State University Fullerton (CSUF) is based on ideas typical of undergraduate learning assistant roles. Colleges and universities are increasingly using learning assistants (LA) to support students in developmental courses with high failure rates (e.g., high proportions of D, F, or W grades). These include bottleneck courses, which have high enrollment but also high failure rates, and are obstacles that delay students' academic progress. Developmental mathematics courses are courses which are designed to help students develop foundational skills necessary for success in college-level coursework. For mathematics courses, learning assistants are undergraduates, typically those majoring in science, technology, engineering, or mathematics (STEM), who work with faculty to help students succeed academically. The type of LA necessary is influenced by factors such as the course's delivery methods (e.g., synchronous or asynchronous, traditional lecture, or flipped classroom), faculty needs, and LA availability (Chaves et al., 2013). Most learning assistant roles take the form of Supplemental Instruction or Embedded Tutoring.

Supplemental Instruction (SI) Program. Supplemental Instruction (SI) was developed by the University of Kansas City in 1973 to enhance student success in traditionally lower-performing classes. Key participants in such a program are SI leaders and students. SI leaders are typically high-achieving students who have successfully completed the associated course (with a grade of B or better) and possess strong communication skills and a desire to help others succeed (Anfuso et al., 2022; Bowman et al., 2023). Typically, SI leaders attend all associated classes, take notes, complete homework, and read the text (Hurley et al., 2006), often adopting a more passive role during class time.

Outside of class, SI leaders conduct special sessions that students can attend, voluntarily. These sessions incorporate active learning-style work designed to reinforce the content delivered previously by the instructor (Hurley, Jacobs, & Gilbert, 2006). Unlike traditional tutoring models, SI leaders do not provide rote answers to student's questions; instead, they redirect and reframe questions resulting in more interactive lessons that promote rich content discourse among students (Anfuso et al., 2022, Bowman, Preschel, & Martinez, 2023).

Embedded Tutoring. Embedded tutoring shares principles with Supplemental Instruction (SI) but differs in key ways. Embedded tutors actively participate in class, posing questions and providing just-in-time remediation, while SI leaders engage with students only outside of class. Embedded tutors collaborate closely with their associated instructor, whereas SI leaders operate more independently, creating SI sessions based on the instructor's plan. Both roles model ideal student behaviors, but embedded tutors emphasize asking rich questions and play an active role in facilitating group discussions during class (Channing & Okada, 2020; Chaves et al., 2013). Embedded tutoring offers a major advantage by integrating student support directly into the classroom, reducing the need for students to seek help outside of class. Students may see visiting a tutoring center or finding extra time for sessions as barriers to be overcome (Duffy & Burkander, 2024).

With the huge shift to online learning post-COVID, embedded tutors have become essential in scaffolding active learning in online college mathematics classes. With synchronous courses, embedded tutors facilitate chat discussions and work with students in breakout rooms for individual and group activities. With asynchronous courses, embedded tutors' model effective online engagement, such as crafting well-composed posts and interacting with peers, while also offering both synchronous and asynchronous tutoring (Channing & Okada, 2020).

Like SI, embedded tutors do provide support outside of class in both online and face-to-face settings. However, this support is usually more flexible and resembles traditional tutoring, yet it still fosters deeper engagement with the course content presented during class (Channing & Okada, 2020; Chaves et al., 2013).

In some ways the apprenticing in the TMAP program is similar to the above supplemental and embedded tutoring forms of learning assistantships. However, in the TMAP program, the work of apprentices differs in that they gradually take on the responsibilities of an instructor. The program is designed to give pre-service secondary teachers and graduate students the opportunity to engage in teaching practices before officially entering the profession. Apprentices integrate into an active learning classroom—both online and face-to-face—at their own pace, working alongside a mentor faculty member. Depending on the mentor's planning and the apprentice's skills, the apprentice's duties may include taking attendance, creating warm-ups and icebreakers, planning and delivering mathematics lessons, sending emails and announcements, facilitating group work, modeling effective classroom interactions, completing observation logs, holding office hours, and light grading (see Table 1).

Table 1. Titan Mathematics Apprentice Program (TMAP) Overview

Timeline	**Program Components**	**Apprentice Participation**
Recruitment Phase	Host informational webinars, in-class announcements, orientation events.	Attend recruitment events and apply for the program.
Selection Phase	Review applications based on GPA, letters of recommendation, statement of interest, and group interview.	Participate in the application and interview process.
Training Phase	Pair apprentices with mentor faculty members.	Begin participating in classroom activities, such as taking attendance and planning lessons.
Initial Engagement	Introduce apprentices to classroom activities and responsibilities.	Meet and work with assigned mentor faculty.
Active Participation	Monitor and support apprentices as they gradually take on more teaching responsibilities.	Engage in teaching practices, including delivering lessons and facilitating group work.
Feedback and Reflection	Conduct regular feedback sessions and professional development activities.	Receive feedback and reflect on teaching experiences to improve skills.
Ongoing Support	Provide continuous mentorship and access to teaching resources.	Utilize available resources and support for teaching activities.
Completion Phase	Assist apprentices in transitioning to student teaching.	Complete the program and prepare for student teaching.

Recruitment and Selection Process. Potential apprentices are recruited from a pool of undergraduate and graduate students studying mathematics who are interested in teaching at the secondary or college/university level. Throughout the year at CSUF, several recruitment activities occur including informational webinars, targeted in-class announcements, and pre-semester orientation events. Faculty will often nominate potential candidates, and students who have had an apprentice in their classes frequently inquire about how they can participate. Applicants are selected based on their undergraduate GPA, letters of recommendation, statement of interest, and performance in a group interview.

Recruitment and retention are also often dependent on securing funding. Duffy and Burkander (2024) suggest:

> Colleges need to develop strategies and incentives to encourage faculty to recruit tutors and provide ongoing support for embedded tutors. They further need to allocate funding to provide competitive compensation and make tutoring positions desirable (p. 9).

At CSUF we are active in our pursuit to secure funding. TMAP is supported by the Mathematics Department, CSU Chancellors' Mathematics and Science Teacher Initiative, and the National Science Foundation (NSF DUE No. 2243558) Noyce Scholarship program, *Recruiting and Transitioning Math Majors into Teaching* (TMMT 2.0).

Mentorship Through Professional Development

The TMAP program offers apprentices a wide range of professional development activities each year. These activities occur both informally and formally. Informally, apprentices participate in professional learning communities and weekly check-ins. Formally, they engage in weekly meetings with their mentor instructors, which can range from discussing plans for the week to co-planning lessons. All apprentices are invited to attend monthly Zoom webinars and are required to attend a summer/intersession training workshop.

Dr. Cherie Ichinose, the director of TMAP, brings to the work her experience in secondary education, including expertise in mathematics education, mentorship, and teacher development. While Dr. Ichinose plays a significant role in facilitating professional development, she is supported by mentor instructors who are CSUF faculty from both the mathematics department and the College of Education. Dr. Ichinose reviews professional development activities and updates them regularly, typically on an annual basis, to incorporate feedback from apprentices, mentor instructors, and the current literature. These updates ensure the activities remain relevant and effective. The professional learning activities are hosted on the university's learning management system (Canvas), facilitating easy updates, collaboration, and accessibility for all apprentices.

TMAP has monthly webinars that focus on issues related to equitable mathematics teaching. These include the series *5 Strides on the Path to Math Equity* (https://equitablemath.org/). In 2024, Dr. Antonette Linton, co-PI on TMMT 2.0, began a series focused on Just, Equitable, and Inclusive Education (JEIE) with a particular emphasis on epistemic practices in mathematics teaching. Among these practices is mathematics literacy, and how it can be translated to social media literacy, with an emphasis on creating classrooms that are socially and academically safe for learning.

In the Fall 2024 semester, six of the senior apprentices, along with Dr. Ichinose, examined the Thoughtful Classroom Teacher Effectiveness Framework, which covers effective teaching, instruction, and professional practice (Silver et al., 2010). Using this framework, they engaged in a quasi-lesson study. Four apprentices were placed in College Algebra and two in a Calculus co-requisite course. Collaboratively, each group developed and executed their designed lesson. After the students executed the lesson, they solicited feedback from their mentor instructor and students. Then the apprentices together reviewed the lessons and feedback and made suggestions for future revision and delivery.

Weekly Zoom check-ins throughout the year are informal, allowing apprentices to guide the discussions with questions on topics such as classroom management, grading, and student engagement. These check-ins sometimes lead to targeted advisement for the apprentices. Additionally, summer and intersession workshops focus on pedagogies that support active learning in both face-to-face and online settings. For example, during the *Be Present—Not Perfect* Summer Virtual Workshop, apprentices were trained on using learning management system communication tools, threaded discussion boards, VoiceThread, and Zoom breakout rooms.

Mentor Matching of Apprentices with Instructors and Courses

Mentor instructor willingness to take on an apprentice and the nature of their classroom environment are the foundation of mentor selection. TMAP specifically targets active learning classes, particularly those that use the flipped class model, where most class time is dedicated to discussion and group work. This approach aligns with the goals of providing apprentices with a dynamic and interactive teaching experience. In these active learning environments, apprentices can observe and practice effective teaching strategies that promote student engagement and collaboration. This hands-on experience is crucial for developing their skills in classroom management, lesson planning, and facilitating group activities, ultimately preparing them to become effective teachers. TMAP supports both face-to-face and large online courses that use the flipped classroom model. With the addition of apprentices, instructors can provide students with one-on-one and differentiated instruction, even in larger sections. The flipped classroom allows apprentices to offer additional support to students who benefit from extra guidance.

Mentor-apprentice matches are based on recommendations from instructors, the availability of classes, students' course schedules, and interest in specific topics (e.g., algebra, calculus, statistics). Instructors may decline requests and may opt-out of participation in the program. Often, mentor instructors will request specific apprentices. Chaves et al. (2013) suggest that apprentice-mentor instructor pairings are often more successful when the apprentice and mentor instructor already know each other, and the apprentice is familiar with the course.

Apprentice Experiences in the Workforce

The success of the apprentice program can be measured by employment records and apprentices' reported perceptions of being better prepared for their student teaching. The mathematics department tracks our students, this includes out apprentices, as they participate in the credential program and throughout their induction program. We maintain regular contact via email and invite them to participate in induction activities such as Math Teacher Circle Webinars. All apprentices, to date, have secured a teaching position within six months of completing their credential and/or master's program. Furthermore, all have expressed that their participation in the Titan Mathematics Apprentice Program significantly contributed to their success. One note: several former apprentices have become mentor teachers for the credential program, further demonstrating the program's impact and success.

Testimonials

The success of TMAP largely depends on the contributions of faculty and students at CSUF. Their voices are central in the ongoing professional conversation that is the program. Participating mentor instructors and apprentices are surveyed to gather their thoughts and experiences. Their insights are crucial in documenting the impact and success of TMAP.

Faculty Testimony.

> *I wanted to take a second to let you know that* ***Apprentice A*** *is absolutely amazing. She has gone above and beyond during these last 12 weeks-I am so thankful for her assistance with everything. She has been such a huge help during our Zoom meetings and announcements all while staying professional and courteous with me and our students. . . . I know how tempting it can be to just explain everything yourself but she let the students be the star. She is going to become a phenomenal educator one day.* —**Faculty A**

Thank you so much for allowing me to work with our amazing future math teachers. Not only did they attend class twice a week, we met weekly for an hour to discuss the upcoming week, supported me with grading assignments, quizzes, and several exam problems – learning about applying rubrics fairly to written student work as well as student Flipgrid videos. They often stepped into the teacher role during class, asking great questions, modeling to students what mathematical reasoning looks like. They learned what collaboration among teachers can look like by forming our Professional Learning Community together. The students were all so expressed, acknowledging how great it was to have a fellow CSUF Titan in class. **—Faculty B**

My apprentices are a dream come true! They are basically my team of consultants for all things student-centered. The apprentices provide me with invaluable feedback on what works best for students. They helped me to clarify tasks. They helped me to organize my Canvas page in a way that is student-friendly. They keep me in check with deadlines and expectations. They truly feel like colleagues and I am so grateful for their contribution. Without a doubt, I am a better teacher as a result of working with them. **—Faculty C**

As the instructor of a large section, I greatly value the presence of the Apprentices. I strongly believe that their contributions are essential to the success of the course. The apprentices interaction with students in small groups of 30 allows for more personalized attention and deeper engagement with the material, especially in a virtual environment. By facilitating the daily group activities (via breakout rooms), the apprentices help foster a collaborative learning environment. The additional office hours they offer are invaluable, providing students with extra support and opportunities to clarify their understanding outside of class hours. The apprentices also plays a crucial role in lesson planning and classroom management, and their assistance in answering questions—whether in person or in the Zoom chat—ensures that all students feel heard and supported. Their presence significantly enhances the overall learning experience for both the students and myself as the instructor. **—Faculty D**

Apprentice Testimony.

I learned that teaching is something that requires a lot of patience and excitement towards the material. ***Faculty E*** *always tried to make learning math a fun process. She always came to class smiling, she made sure that anyone's questions are always welcomed. Working in groups with the students and helping them helped me grow as a math educator. It's important for me to have tutoring opportunities and* ***Faculty E*** *shadowing for my mathematics educator growth. I have tutored before, but it was a part of AVID where students come with questions however, being an embedded tutor was much more beneficial. I don't really know what more opportunities to get when being a mathematics tutor. I really liked what I did this semester!* **—Apprentice A**

I learned a lot of pedagogy and classroom skills, especially when working with students who have a negative view on math. I feel I gained a lot of insight and new perspectives through our monthly webinars. **—Apprentice B**

*TMAP influenced how I thought about teaching. I was always a Math major with a concentration in teaching but apprenticing **Faculty F** gave me great practice with working with university students. Working alongside **Faculty F** in planning on how instruction will be presented gave me an experience that still has an influence with how I run things in the classroom that I currently student teach in.* **—Apprentice C**

*This semester I was an apprentice for our introductory statistics class. This opportunity helped me better prepare for my future classroom. I plan on teaching statistics for high school students, so apprenticing for college statistics is perfect practice. I loved the students this semester too. **Faculty G** gave me plenty of free reign on my responsibilities in the class.* **—Apprentice D**

Before joining the program, I didn't have any teacher role models in my life . . . my parents aren't teachers (and) the only teachers I saw in my life were the ones that were teaching me as a student, and that the TMAP program provided me valuable examples of how to be an effective teacher. **—Apprentice E**

*This semester, I learned a lot about how to best challenge and help students with their problem-solving skills. **Faculty H** did a great job of asking the students questions about their work, particularly asking them to give reasoning for the steps they make in the problem-solving process. This inspired me to ask the students similar questions, often asking them "Why did you do that?" when they would explain their reasoning to me. One insight I gained this semester is that sometimes students do not know why they are taking certain steps in the problem-solving process, yet they do it without asking the instructor why. I realized that we, as instructors, need to ask our students "why" to challenge them to think more critically about what they are doing and grow in their conceptual understanding of the material.* **—Apprentice F**

Conclusion

The Titan Mathematics Apprentice Program (TMAP) at CSUF has successfully prepared future mathematics educators with practical experience, mentorship, and professional skill development. Key recommendations for similar programs include establishing a clear recruitment and selection process through webinars, announcements, and a thorough application, interview, and review process. Further, providing comprehensive training and ongoing professional development via workshops, webinars, and regular check-ins is also crucial. Another important aspect is the matching of apprentices with mentor instructors in active learning environments. Securing funding from institutional and external sources is vital for sustaining the program. This funding is primarily allocated for course release for the director, summer salaries for the principal investigator and co-principal investigator for the Noyce program, TMMT 2.0, and much of the funding is dedicated to the salaries of the apprentices. Finally, maintaining contact with former apprentices, tracking their progress, and inviting them to ongoing professional development activities helps ensure their long-term success as educators.

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About the Author

Cherie Ichinose is a Professor of Mathematics at California State University, Fullerton, where she coordinates the Single Subject Credential Program in Mathematics and directs the Titan Mathematics Apprentice Program. Her research examines the role of emerging pedagogies and technology in mathematics education, including flipped classrooms and online learning. Currently, her work focuses on how Project-Based Learning (PBL) shapes future secondary teachers' perceptions of teaching Statistics and Data Science. She has actively contributed to CSU-wide initiatives, presenting at Professional Learning Community meetings, Summer Learning Institutes, and system-level gatherings with Presidents and Trustees.

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Measuring the Impact of the Flipped Model in a Pre-Calculus Course

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Abstract. The flipped model of instruction has been championed as a method for increasing student engagement and enabling instructors to facilitate more active learning within the classroom. Research on the flipped model also suggests that it may also lead to better course outcomes than the traditional lecture approach. Given this potential for student success and engagement, faculty at California State University, Fullerton have adopted the model in many of their courses. In this preliminary report, we measure the impact of the flipped model on course outcomes of a Pre-Calculus course during the Spring 2024 semester. In addition, we compare student perceptions of the course elements that supported their learning between students enrolled in the flipped sections of the course and those in the non-flipped sections of the course.

Introduction

The flipped model of classroom instruction is a mode of active learning rooted in the constructivist theory of learning (Bishop & Verleger, 2013). In this model, students consume course content outside of the classroom through video lectures and/or course readings and spend in-person classroom time engaging in collaborative learning and problem solving (Lage et al., 2000; Salifu, 2017). Since gaining popularity during the early 2000's, various scholars have championed the model for its effect on student achievement (e.g., Caviglia-Harris, 2016; Connell et al., 2016; Day, 2018; Le & Ichinose, 2022) as well as attitudes towards learning and classroom engagement (Alebrahim & Ku, 2020; Stone, 2012).

Within the past decade, there has been a great shift towards active learning pedagogies (Stone-Johnstone et al., 2024) within the mathematics department at California State University, Fullerton (CSUF). One of the most popular active learning approaches used at CSUF is the flipped model, with many introductory mathematics courses (e.g., College Algebra, and the Calculus series) being flipped by certain professors. In this preliminary report, we evaluate the effect of the flipped model within the Pre-Calculus course at CSUF during the Spring 2024 semester by answering the following research questions:

1. Is there a statistically significant difference in course outcomes between the students in a flipped vs. non-flipped Pre-Calculus course at CSUF?
2. Are there differences in student perceptions of the course elements that supported their learning between the two groups?

Methods

Context. The flipped model, as employed at CSUF, is a three-part model that consists of online interactive content videos, pre-assessments (Ticket in the Door, TITD), and mathematics labs. Traditional lecture time is reconceptualized within this model as a mathematics lab, where students work collaboratively on activities in their course workbook during in-person class time. Data for this study was collected during the Spring 2024 semester, where there were eight coordinated sections of Pre-Calculus with an average of 35 students in each section. Out of the eight sections, four were flipped and the other four were non-flipped. In the non-flipped sections, students primarily learned course content within the classroom, with some instructors choosing to post optional resources for students to engage within the learning management system (Canvas). All students were assessed using common midterms and final exams.

Theoretical Framework. Given the emphasis on student classroom engagement within the flipped model at CSUF, we view the data through the lens of the Instructional Triangle (Cohen et al., 2003; Lampert, 2001). The instructional triangle is a framework for articulating the process of teaching and learning. This triangle consists of three primary nodes: student, teacher, and content (see Figure 1). The arrows represent the interactions between the nodes. As Cohen et al. (2003) explained, "instruction consists of interactions among teachers and students around content, in environments" (p. 122).

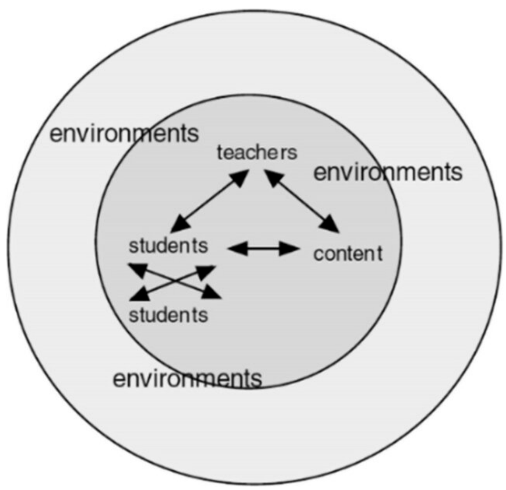

Figure 1. The Instructional Triangle (Cohen et al., 2003).

Student-student interactions may occur through in and out-of-class collaboration. Student-teacher interactions include teacher moves for engaging students and supporting their learning (Ball & Forzani, 2009). Student-content represents how students engage in course content, and this may be through course resources (e.g., homework, videos, textbook). Teacher-content interactions represent the instructional choices for content delivery. The concentric circles within Figure 1 represent the varying levels of the environment in which this learning transpires (in and outside the classroom). Given the differing pedagogical approaches within the flipped vs. non-flipped courses, the instructional triangle enables us to explore the student experience by examining the types of interactions that students identified as contributing to their learning.

Data Collection. This study was conducted with approval by CSUF's Institutional Review Board. There were two primary data sources employed to address the research objectives: student course outcome data for all Pre-Calculus students during the Spring 2024 semester and survey data. The authors constructed a Qualtrics survey, consisting of three parts. The first part consisted of Likert-type questions, gauging student perceptions about course structures (e.g., course workbooks, activities, and assessments). The second part consisted of three open-ended questions regarding their perceptions of support and barriers to their learning of course content. The final section consisted of demographic questions including information about previously attempted mathematics courses, placement scores (for entrance at CSUF), and their projected course grade.

Data Analysis. Employing a mixed-methods approach to data analysis (Creswell, 2012), we answer the first research question by conducting a quantitative analysis. Specifically, we performed a two- sample $t-$test to determine whether there was a statistically significant difference in the course outcomes between flipped and non-flipped courses. Students who withdrew from the course ($n = 25$) were removed from the analysis since no point value was assigned to a grade of W.

We answer the second research question regarding student perception of the support for their learning of Pre-Calculus, by conducting a qualitative analysis. All Pre-Calculus students were asked to complete the Qualtrics survey during the ninth and tenth week of the semester. Of the 276 enrolled students, 173 completed the survey (approximately 63% response rate): 88 students in the flipped course and 85 in the non-flipped course. For the purpose of this investigation, we focused on students' responses to the open-ended question, "What are two or three components of the course that have supported your learning?"

All student responses were coded using a coding scheme adopted from the Instructional Triangle framework (Cohen et al., 2003). Since students discussed aspects of the course that they found beneficial to their learning, we limited our coding to focus on the Student-Student, Student-Teacher, and Student-Content interactions within the Instructional Triangle. The dataset did not allow for us to explore the Teacher-Content interactions, nor teacher motivations for their instructional choices (i.e., Herbst & Chazan, 2012). Throughout the coding process, a subcode of the Student-Content interaction was created, "External Resources," to describe instances where students relied on learning resources separate from the ones provided by the instructor. A second subcode, "Office Hours," was generated to differentiate between in-class teacher support and outside-class teacher support, given the additional effort required for the latter.

There were 132 complete student responses to this open-ended question. The first author coded the entire dataset, and 30% of the dataset was double-coded by two other members of the research team. Instances of disagreement were discussed and negotiated, with only two instances of disagreement (95% agreement).

Preliminary Findings

Course Outcomes. The result of the two-tailed independent samples $t-$test was not significant based on an alpha value of .05, $t(248)= 1.53$, $p = .126$, indicating there was no significant difference in the course performance (course grade point value) between the flipped and non-flipped classrooms. The results are presented in Table 1. While the difference in performance was not statistically significant, the overall mean score in the flipped classrooms was slightly higher. In particular, the mean value of course grades is 0.27 points higher for the flipped modality compared to the non-flipped modality (1̃3.6% difference).

Table 1. Two-Tailed Independent Samples t-Test for Final Grade Point Value by Modality

	Flipped			*Non-Flipped*				
Variable	M	SD	*n*	M	SD	*n*	*t*	*p*
Point Value	2.25	1.31	114	1.98	1.36	136	1.53	.126

Perceived Supports for Learning

When comparing the students in the flipped course to those in the non-flipped course, we did not observe meaningful differences in the course components that students identified as supporting their learning. Table 2 illustrates the frequency of the different codes across the two groups of students.

Table 2. Distribution of Code Frequencies between Students in Flipped and Non-Flipped Courses

	Flipped	*Non-Flipped*
Student-Student	11	5
Student-Teacher	27	22
Office Hours	1	8
Student-Content	57	50
External Resources	2	5

More of the flipped students mentioned Student-Student coded elements as supporting their learning, with one student commenting: “We help out what questions we have to answer as a group while working on the workbook and worksheet.” Some students in the non-flipped course also expressed valuing group collaboration, explicitly naming “in class group activities” as supporting their learning.

Students in both groups credited their instructor’s in-class explanations and lectures for supporting their learning. However, it is worth noting that eight students from the non-flipped group (compared to just one from the flipped group) specifically mentioned attending their instructor’s office hours as a course element that supported their learning.

The majority of the dataset was coded as “Student-Content,” with both groups of students identifying the workbook, the online homework, and, for the flipped classroom students, the Ticket in the Doors as valuable to their learning. The primary differences that arose between the two groups were with respect to the types of external resources students credited. Two flipped students mentioned workshops (Supplemental Instruction) as a support, while the five non-flipped students mentioned resources such as YouTube and Khan Academy.

Discussion

The preliminary findings revealed no statistically significant difference in course outcomes between the students in a flipped vs. non-flipped course; however, the data does suggest the flipped model may still have potential benefits for course outcomes. The observed 13.6% higher mean point value of letter grades between the flipped and non-flipped students is promising.

Similarly, we did not observe notable differences when comparing the student's perceptions of course supports between the flipped and non-flipped modalities. Students from both modalities primarily credited their learning to course elements such as homework assignments and in-class workbook activities.

Considering the active teaching culture within the mathematics department at CSUF, one could speculate that the non-flipped model might have incorporated more interactive elements compared to a traditional lecture-based course. If so, this could account for the similarities in student perceptions regarding the course elements that supported their learning between both modalities. The only notable difference in students' perceptions was that non-flipped students reported using Khan Academy and YouTube for video lecture support whereas the flipped students already had instructor-provided videos embedded within the course.

As we continue to explore this data, and the flipped classroom model in general, we invite the audience to consider the following questions:

1. Though we found no significant or noticeable differences between flipped and non-flipped modalities of instruction, how does this finding contend with the current teaching and learning climate?
2. What specific aspects of the flipped and non-flipped course formats might have led to the observed lack of statistically significant difference in student outcomes?

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Dreams from the Past: A Review of *California Dreaming*

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Abstract. This is a review of *California Dreaming: Reforming Mathematics Education*, by Suzanne M. Wilson. The book describes the efforts to change the mathematical standards for K-12 in California schools, primarily in the 1990s. The reviewer explores the content of the book and describes who might benefit from reading the book.

California Dreaming: Reforming Mathematics Education, is the story of efforts to change K-12 mathematics teaching in the 1980s and early 1990s. As Wilson explains in the preface, "I tell this story neither to demonize one or another of the involved groups, nor to oversimplify the events" (p. xii). The book then takes a mostly chronological approach, delving into earlier history a bit to set the stage, and then describing the key people, processes, ideas, and events that led to major changes in the state curriculum and associated assessments, first towards reform, and later away from it.

Wilson helpfully describes the structure of the California state education bureaucracy. There is the governor-appointed State School Board of Education (SBE), the California Department of Education (CDE) and its elected head, the state superintendent of instruction, and the group known as the Curriculum Commission (CC), with 13 of its 18 members appointed by the SBE. The CC writes and revises the curriculum subject frameworks with assistance from CDE staff members. Ultimately, the SBE has the authority to adopt a framework submitted by the CC. This is only the beginning, as there is subdivision of the work of the CC and public comment periods during development. In addition, K-8 textbooks are approved by the state in a complex adoption process, and the state has its own testing regime. Wilson provides diagrams illustrating some of the relationships and processes, including one of the interacting state agencies and one of the textbook adoption process.

Wilson explains the lines of debate as well as the terms. She expends considerable effort explaining the terms used by the different sides in arguments over curriculum and pedagogy, for instance, "constructivism" (p. 39) and related terms are discussed in the context of the 1985 Curriculum framework. She highlights the tension between what is taught and how it is taught, as well as tensions within the community that wrote the 1985 framework.

Wilson also provides some key insights into the thinking and the limitations in thinking of different players in the system. For instance, she notes that Bill Honig, elected State Superintendent of Instruction in 1982, targeted changes in the formal bureaucracy, but that the larger context in which this system exists was essential to the success or failure of efforts at reform. Appropriately, then, Wilson also details some of the key individuals and organizations and their impact on mathematics education, including Marilyn Burns, Judy Kysh, Judy Mumme, Phil Daro, the California Mathematics Council, and the California Mathematics Project, CMP (disclosure: the author is and has been the Director of a CMP site for 10 years). Later, as the story shifts to the unraveling of the reform, she discusses groups such as Honest Open and

Logical Debate (HOLD) and the website Mathematically Correct, as well the criticisms from folks like Henry Alder and Hsung-Hsi Wu.

While the focus is on California, Wilson does portray the interaction between the changes happening in the state and similar debates and efforts at the national level. The national context includes reports like *A Nation at Risk* from 1983, National Council of Teachers of Mathematics documents, and the movement of key individuals between California and national efforts.

For perhaps half of the book, the story is about the rise of reforms in California, and then it shifts into the rise of its opposition, beginning with criticism of the California Learning Assessment System (CLAS). Wilson is helpful in citing the ways in which criticisms arose louder and louder, with "media coverage [that] inflamed the discourse with strategically selected examples that fanned passions rather than furthering deliberation," (p. 134). Wilson cites potential contributing factors to perceptions of poor testing performance that were largely ignored in debates, such as California's decline in comparative per pupil spending, from fifth among the states in 1964-1965 to forty-first in 1994-1995, and the fact that for a majority of California school students, English is not their first language. Whatever the contributing factors, opposition to testing culminated in a veto of the CLAS reauthorization bill by Governor Pete Wilson in September 1994. Later, there is a discussion of the many overlapping and conflicting values at play in discussions of public education, and the undercurrents, such as racism, that also contributed to the conflict. Wilson notes the tendency of some on both sides toward dogmatism, and the consequences:

> It seems reasonable to assume that some people encountered a questionable constructivism, and others–when they raised questions–felt dismissed. Meanwhile, some progressive educators also felt that they were encountering an oppressive traditionalism. Eventually, the either-or dogmatism that some critics and reformers encountered would exacerbate the "war," heightening passions on every side (p. 151). Late in the text, she points out that "the story has no end" (p. 202). Throughout, Wilson is mindful of the difference between changes in policy, curriculum, and testing, on the one hand, and actual classroom teaching on the other, and she eventually shifts to a discussion of what classroom instruction was like at the time, and some of the forces that mitigate against change.

Who would benefit from reading this book? Many of us who work in or alongside K-12 mathematics education or who are interested stakeholders, we who are not familiar with the bureaucratic structures that govern educational policy, the history of reforms, their impacts, and the reactions, would benefit. This book provides insights on just those issues. I first read the book in 2003 as a relative newcomer. As I write this in 2023, the newest California Mathematics Framework has been delayed multiple times as a result of public criticisms of the draft. Are we merely repeating history, or is there progress, as the lines of debate shift? That is a discussion for future historians. For now, consider spending time with California Dreaming, and you can draw your own conclusions about where we go from here.

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