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Bringing Joy to Uninspired Teachers of Math
Touchstone Strategies, Part 3†

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Abstract. The third in a three-part series, this resource describes and illustrates three of eight Touchstone Strategies for teacher educators to use in their work with mathematics teachers. The article explores how to inspire teachers to find the joy in mathematics so they can support their students to do the same. Through a variety of tools, techniques, and helpful hints, the eight touchstone strategies in the series illustrate what high quality mathematics instruction looks like and how teachers can reframe their own thinking about mathematics to create deeper learning opportunities for their students. This piece, Part 3, describes the three touchstone strategies: *work a problem "to death," non-dominant language*, and *concept teaching games*.

Touchstone Strategy #6: Work a Problem "To Death"

Work one intentionally perplexing problem "to death" unearthing confusions that arise.

I have found it beneficial to provide teachers repeated opportunities to deeply solve a confounding problem in a collaborative group, in order for them to see that it can and should take time. Each semester, I would give people two to three classes to work on what I called "The Letricia Problem" (mentioned in earlier parts of this article and described below) — probably the most eye-opening task I have ever watched teachers work on together. During those three weeks, we intersperse concept games related to measuring and geometry, which are topics embedded in the Letricia Problem. In addition, we do some Number Talks and have a few text-based discussions on articles about the conceptual roots to the geometric and measurement ideas embedded in the problem.

If teachers actually live through a productive and enlightening problem-solving process themselves, I believe they are more likely to become courageous enough to offer the same eye-opening experience for their own students. They tell me "doing" math like this in my class is their linchpin since most rarely have done that before enrolling in graduate school.

After reading and discussing the chapters in Making Sense (Hiebert et al., 2000), particularly the ones about the chapter focusing on the social culture of a progressive math classroom, they are more than primed for this experience. They read and we discuss how a group of four girls in

† From an original report by Melnick (2018), this is the third of a three-part reprint with the permission of the author. The full report is available from the Bank Street College of Education website.

fourth grade struggle to communicate when solving a division of fractions problem (Using 20 apples, how many apple tarts can be made if it takes $\frac{3}{4}$ of an apple to make just one tart?) After listening to kids working through their different perceptions and conjectures about solutions, the class is ready to mirror the same process.

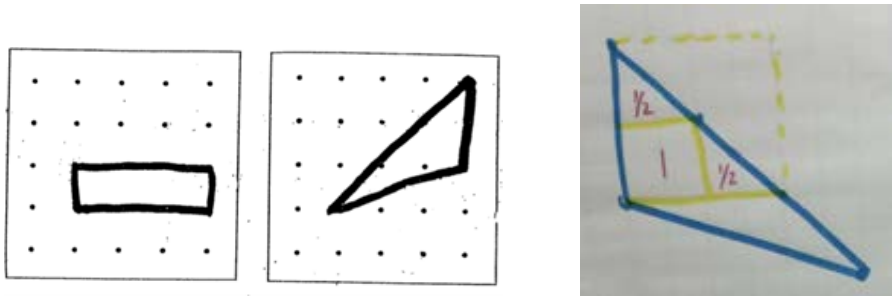
My work with collaborative groups starts with forming groups with people of varied skills and needs (re: style of communicating, feelings about math, potential learning disabilities, varied pace of learning needs, as well as the age they teach). Have them solve a complex problem by showing at least two different strategies, including one that uses concrete tools while abiding by Burns' "Three Rules for Small-Group Work" (see Touchstone Strategy #3 in Part 2). This is the time for us to "walk the talk." I teach as I expect them to teach. This is the moment for theory to intersect practice. We learn by doing. They need to collaborate and I need to stay out of their way.

This Perplexing Math Problem solving event is an opportunity for them to practice the skills they are developing as math teachers: listening, giving hints to others without giving answers away, honoring different styles, different paces, not being overbearing in pushing an agenda on others, being open to new and different ways to think about the whole problem you are working on, being interactive. It is an opportunity for each of them to be a good teacher to one another. I insist upon multiple representations, which was one of the five NCTM's Process Standards in NCTM's Curriculum and Evaluation Standards for School Mathematics (1989). I tell them to use drawings, arrows, equations, words, and cutouts depicting their visual thoughts. I do so with the express purpose of enhancing equity and accessibility. I model one way to undercut overvaluing the so-called 'brightest' or 'fastest' learners who could otherwise try to monopolize class time and brag about their erudite solutions.

The genesis of the key problem I use, called the "Letricia Problem," came about when I was working with doubting teachers of grades three to five in one public school where I was consulting. They each claimed that they had shelves filled with wooden geoboards in their classrooms sitting there for years and years, but no one saw a use for them. They also doubted my suggestions that heterogeneous cooperative learning groups could magnify learning for all and could possibly satisfy children's needs across varied levels of sophistication. They challenged me to prove to them that the geoboard could be used to achieve the goals I had put forth for math learning, including those embedded in the state standards for their grades.

On the spot, I made two shapes on one geoboard. One was a 3 x 1 rectangle and one was a scalene triangle. See Figure 1.

Figure 1. Two shapes on a geoboard and triangle close-up



I thought they were about the same size in area but I could not at first tell. I knew that it would be easy to find the number of square units in the rectangle, but saw in front of me that the

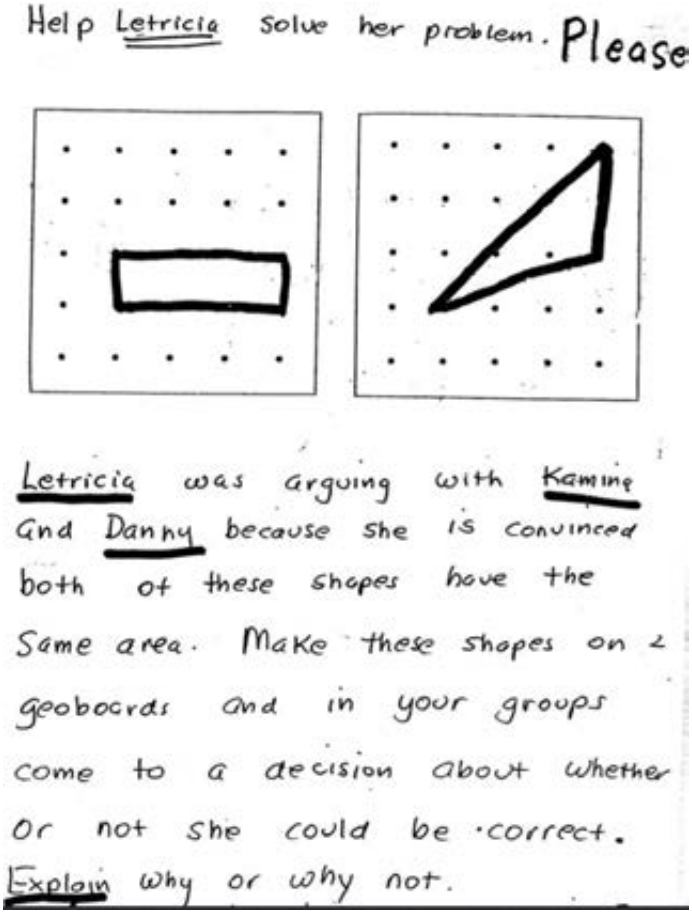
triangle's area would be a bit harder to figure out in square units. I myself did not immediately know how to do it. By using rubber bands I could see there was an easy part to figure out with an area of two square units inside it, but then there was an annoying leftover skinny triangular section that would require some manipulation and use of logic in order to find its area.

I decided to go with these two shapes. I had no idea how many different methods could be used to solve it, but I trusted it could be solved somehow. I went with this in the moment and presented the problem using a classic method I had been taught by Bob Davis when I was a young teacher in the 1970s studying his Madison Project Paradigm Teaching Strategy at Syracuse University (Davis, 1980).

I made up a girl named Letricia and imagined her in a class with two boys called Kamina and Danny. Using Davis' Paradigm Teaching Strategy, I created a hypothetical situation where three kids were arguing or debating the relative merits of a solution or a strategy to a problem. The premise to Davis' strategy is then to ask the class to resolve the children's hypothetical conflict by thinking together using words, numbers, drawings, or equations and, in at least two ways, write, draw, and define your group's solution. This worked like a charm and has been replicated in every one of my 80 or so classes since then.

Figure 2 is the Letricia Problem as presented to students in the class.

Figure 2. Letricia Problem



Below are the strategies that emerged during one section of the Spring 2017 class. I always require at least two strategies. I tell them, by having two irrefutable strategies, they can verify they are right. I also ask people to name the strategies they employed using their own nonconventional language. We post the name of their preferred strategy next to their explanations. On the next pages are images of the strategies that emerged during my last semester of teaching. I have done this problem for at least 20 years of classes and it seems fresh to me each and every time. Geoboards, brightly colored dot paper, scissors, glue, bands, markers, and large poster paper are all provided. Teachers first solve the problem quietly as best they can alone, then they share their conjectures with their group and begin to work together in class. After considerable time to listen to each other and to delegate responsibilities for the final product presentation, they represent their findings using at least two methods.

To help me assess each student's ability to work in a group, I also require a multimedia Perplexing Problem assignment where groups of four present their solutions to one perplexing problem done during the semester. They may choose to represent the Letricia Problem but they also have a list of other challenging ones from which to choose. They are to compose a presentation and write it up together. After designing their PowerPoint or Google Slides presentation, they are each to write: 1) what they learned about math by working collaboratively and 2) what they learned about themselves as a member of a collaborative math group.

The following are some of the insightful responses I feel depict many of the emotional challenges teachers encounter when doing collaborative math together. Read the following with a lens toward the emotional affect. What feelings or emotions are evoked when Bank Street students are asked to reflect on the active process of working within a group whose end assignment they had to submit as a shared entity? Is there any evidence of coping and adaptation? Might they be learning from what they feel after going through this?

> I learned that everyone needs to go at their own pace. Once I got the answer, I wanted to be done, or at least have everyone thinking and feeling the same way as I did, but I realized that some people needed more and it was my job as a team member to give that time. I also learned that problem-solving in a group is REALLY rewarding! I felt like by the end of the time we had really gotten to know each other as learners and teachers. —Abby

> I feel I was being impatient at times. Is it harder to be on the inside as one of the group members or on the outside as the teacher? I felt I needed to keep switching back and forth between the two mindsets. The process is rewarding and challenges you to really listen. Definitely a lesson in patience. Some of the qualities that make us caring adult group members have come with maturity and can't be fully expected from kids who are not developmentally capable. —Alex

> As a member of a collaborative group, I found myself to be patient in the process. This is because I myself am very curious as to how others in my group solve problems. Further, proving that I am someone who builds huge amounts of confidence from having others present in the collaborative process. In fact, because math in general can be a stressful time for me as a learner, having others present makes the process that much easier, making math an engaging experience. —Gabe

> I learned that it is hard for me to deal with the disequilibrium caused by exposure to different approaches and lines of thinking. I experienced frustration at my inability to get team members to take on my perspective. I discovered that my team members were open to letting me go in a different direction, explore my own curiosities about the problem, and share back what I had discovered. —Merry

Each semester I wonder about how long I can devote to one problem. And each semester I am deliriously happy that I have allowed as much time as I did. Not only do I feel I unleash wildly varied thinking in my students, but also a range of mathematical content. We don't wake up each day and do number operations first or geometry at 10:40 AM or algebra at 1:30 PM. When you solve a real life problem, you use all the knowledge you have, and you ask friends to help you because they might have some bits of information and skills you don't have. That is what 'doing math' should look like and the Letricia Problem offers just that. Just doing a quick scan at the photos of the sample from one class, we see how the math discussed included:

(1) discussion of polygonal characteristics,
(2) doing area calculations using formulas remembered or invented,
(3) using doubling and halving,
(4) spatial logical reasoning,
(5) the Pythagorean theorem and exponential work,
(6) trigonometric work with opposite angles, and finally
(7) aspects of high school motion geometry using "flips, slides, and turns" otherwise called reflections, translations, and rotations respectively.

All this is possible with just one problem. Creative problem solving and strategic competence is developed as well. How much better can a class get?

Figure 3. Sample work on the Letricia problem.

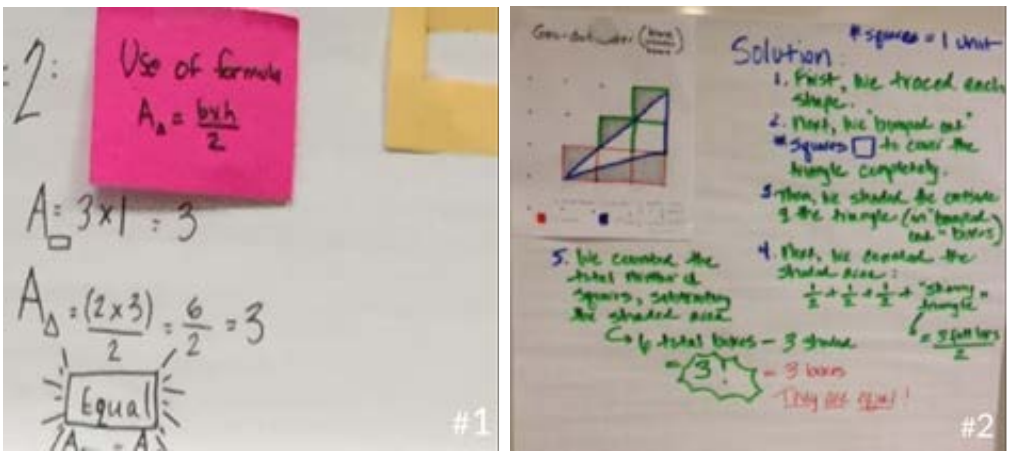
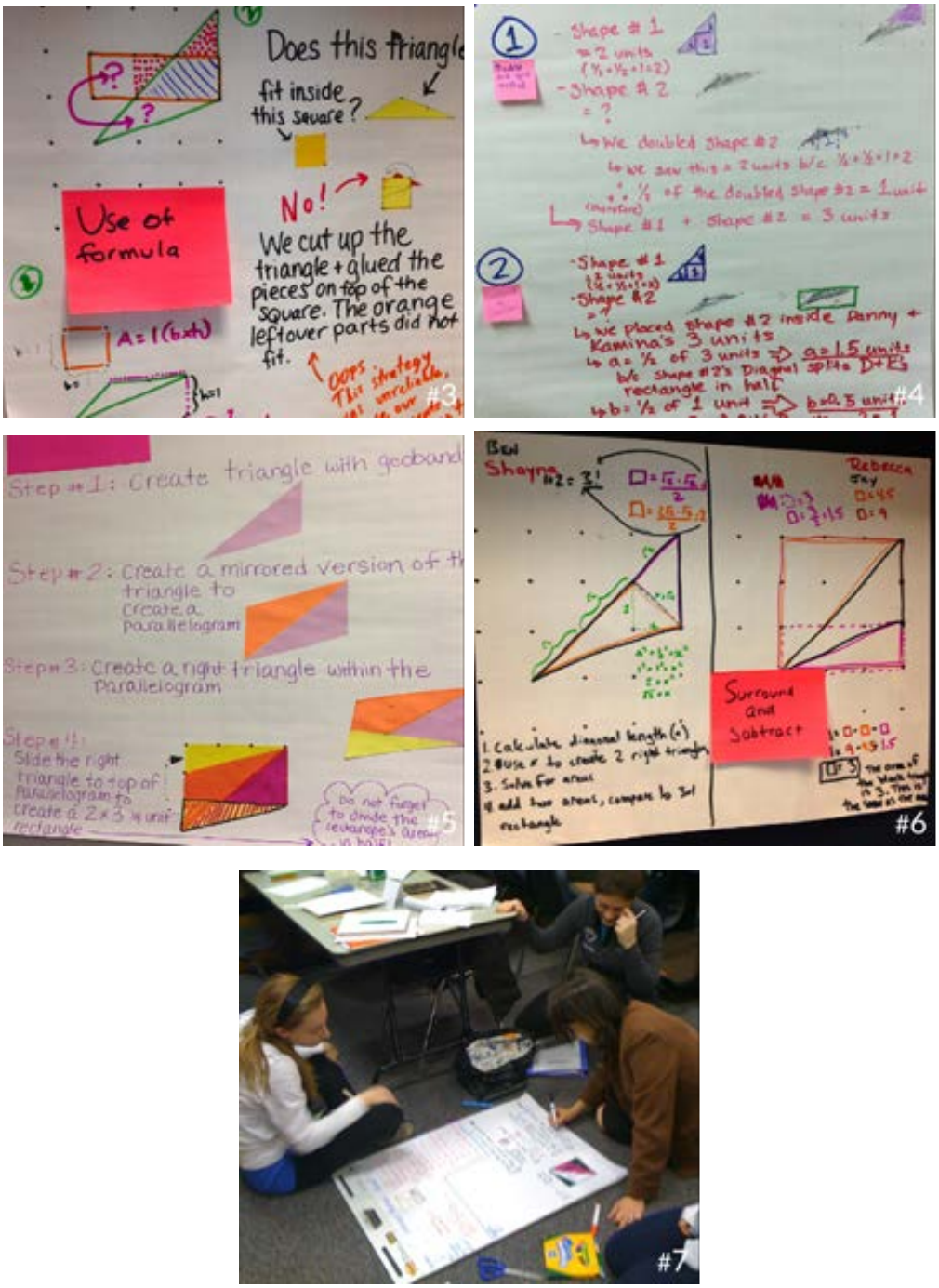


Figure 3. Sample work (continued)



Touchstone Strategy #7: Non-dominant Language

Teach one night's class in a language other than the dominant one.

One of the greatest challenges I have had in teaching at Bank Street has been dreaming up ways to help students in my classes connect social justice issues, equity issues, and discriminatory

practice conversations to the teaching and learning of math. We can have spirited discussions about Geoffrey B. Saxe's seminal study of Brazilian candy sellers, which helps to break down stereotypical views of children who fail math in school. Students are amazed to see how these children failing math in school can, at the same time, run successful candy-selling businesses at the bus depots in Brazilia. It helps graduate students at Bank Street to easily see how real-life mathematical applications can motivate all learners, both in and out of school (Saxe, 1991). That being said, I still find it extremely difficult to evoke an emotional connection between this text and most of our Bank Street students' own lives. The larger percentage of our students are usually working as teachers or assistant teachers in independent schools in the New York, New Jersey, and Connecticut surrounding areas. And the bulk of our students are themselves quite privileged in that they were educated in well-resourced settings and likely neither attended schools in underserved areas nor have they yet taught in such schools.

Of course, there are always those well-versed in such issues. Often these adult learners become frustrated with the lack of awareness of their peers. They generally comment that social justice issues are too seldom naturally addressed by graduate school classmates. In my classes, I used to find they were correct. For most of the students I taught in Math for Teachers, it was often a stretch to connect emotionally with the situation that many children and teachers in inner-city schools face—where supplies are limited, funds are lacking, and quality leaders are equally scarce.

While planning for a discussion about National Council of Teachers of Mathematics' "Spotlight on the Principles: The Equity Principle through the Voices of African American Male Students" (Berry, 2004), which is additionally articulated in *Making Sense* (Hiebert, 2000), I came up with an idea. I was about to start using the geoboard for the Letricia Problem, but I wanted students to become familiar with ways first graders could creatively engage with polygons and geometry first using the geoboard manipulative. I also wanted to demonstrate a way to integrate art and geometry. I decided to teach a classic Marilyn Burns activity: the "Things That Fly" graphing lesson. In her lesson, first graders are asked to stretch a few rubber bands (called "geobands") across nail heads on the geoboards to make something that "flies." They are then to transfer their images (map them) to small pieces of geodot paper and hand them to the teacher to make a graph of "Things We Made on the GeoBoard That Can Fly." This nicely leads into graph interpretation and data analysis for young children. Each person becomes represented on this graph because there is one piece of data drawn by each child. We then discuss questions about more and less data points. After doing this activity, the kids can suggest categories of other things to make on the board (animals, vehicles, etc.), leading to various forms of classification and sorting tasks. In addition, due to the 25 points on the board, some artistic license is required and creative expression is nurtured. Nice task, I thought. Perfect for grade 1. Then it hit me. This is a task that anyone can easily follow and, though it might be a good challenge for a first grader, it is not much of a cognitive challenge for an adult. Because I wanted to marry the notion of emotion in learning math with the notion of privilege, I came up with an idea. Teach this lesson in Spanish! That semester I was co-teaching Math for Teachers with colleague Dr. Olga Romero, who was then Director of Bilingual Education at Bank Street College. We both wanted to teach by incorporating second language learning techniques and decided this task would be a perfect segue for our two skill sets to merge. We decided that Olga could teach this whole lesson in Spanish, modeling the experience of an immigrant arriving from a non-English-speaking country. As we planned the lesson, we both listed the plethora of ELL techniques that could be useful: full body expression; writing key content-obligatory words on cards and introducing each to the students with great expression; placing language peers together and interweaving the two languages during the lesson; showing pictures to attach meaning to words; repeating words over

and over and affirming accurate responses boldly. Multi-gesturing would work well in this lesson. We thought about topics that might emerge for discussion after the whole lesson had ended. Most important, I thought this might bring out emotional reactions for all our students. I was psyched.

The first semester that I planned this, there were some in the class for whom Spanish was in fact their first language. I hoped that would offer them a sense of privilege. Little did we know that we would trigger a huge range of emotions in that class. I remember watching one young woman run out of the class crying and noticing clear annoyance on some faces and expressed body language showing obvious discomfort. Some had broad smiles on their faces.

Debriefing the session immediately thereafter opened the door for strong emotional commentary. I was pleased to hear from Spanish-dominant speakers, who each said, in different words, that for the first time they felt fully at home and fully "in the know." Privileged! Some non-native Spanish speakers were not so happy and expressed their discomfort. Journals after that session were filled with the full range of emotional responses. Some expressed intense anger at me for "wasting" their time. People wrote that they did not come to graduate school to "not understand" class content. This encouraged ongoing discussions regarding equity issues and issues of privilege. The "sturm und drang" that ensued ensured for me that this class session was to become one of the most important ones that I could teach. The varied emotional responses gave rise to deep and meaningful interaction.

For the past 10 years, I have always done this same lesson in Spanish and the responses each semester are strong and powerful. I carefully explain my goals for the lesson to my teaching assistants, many of whom are able to teach the lesson in a nondominant language. Dominant English speakers are offered the experience of being unable to fully understand the discourse, a feeling they may never have had. They are outsiders for the first time. Their emotional selves are challenged to tackle something they absolutely feel unable to do. They might be challenged to use all the resources around them; interacting with their table mates for help in understanding, using visual clues, depending on partners at the table who have language proficiency. Many find the work extremely difficult as they struggle to know something they don't know at first. They become sensitized to the needs of the disenfranchised. Some are enchanted with the challenge and appreciate the experience. And for the Spanish-dominant students, the glee on their faces tells it all. When teaching math, the opportunity to step into language difference is a powerful tool for feeling what "others" feel.

On feeling insecure:

> The class that Concetta spoke only Spanish to us showed me how some students feel in school who experience English as a second language. This made me feel insecure, but by the end of the class I realized there was an important message behind this. I had previously done this in my language acquisition course last semester, but this time it really connected to me. It made me realize that this is how I felt my whole life in math classes. I felt as though with more and more formulas shown to me, the less I understood and the more I felt as though the teachers were speaking an entirely different language to me. It made me understand that as an educator, it is my responsibility to make sure that not only am I teaching, but that the students are learning. There is a huge difference between the two and, often times, it is not regarded. —Erica

On learning math in a language she doesn't understand:

A guest from the second session came in to teach a lesson in Spanish. That had a pretty profound effect on me, and it's something I've been digesting since. I spoke about this a bit in class, but it really registered and I felt a range of emotions that I think are going to better me as a future educator.

The first sensation I had upon the start of the lesson was confusion—as in, what is happening? I then felt an initial twinge of, honestly, annoyance. (I took French growing up, and while my language abilities have disintegrated with time, I could communicate well enough in that scenario had the guest been speaking in French.) But anyway, that twinge of annoyance I felt was based on, "Why are they assuming every person here took Spanish growing up, or in college?" I then switched gears by turning to my classmate, Kelley, to try and figure out what was going on and also looking to others to follow along, which worked well enough for some time.

But when she started writing on the board, I distinctly remember putting down my pen thinking, "I'm out, this one is beyond me." However, when we started that class discussion after, it was a real—pardon my French, no pun intended—holy sh*t moment for me. I had completely missed the point of why we were doing that in the moment, but it struck me like a punch to the stomach afterwards. Seeing what it's like for an ESL student to learn, and how we as teachers need to reach them regardless of their English capabilities, should clearly be a top priority for anybody who might be in that scenario.

The irony is, I've TAUGHT English in Tanzania to Swahili-speaking children, which I've touched on in a journal or two before. I've also volunteered with high school students through a program called Minds Matter, which pairs high-achieving students from low-income families with a mentor, who helps guide them through the college application process and generally prepare them for life beyond. Those students who I interacted with were at that point in age all fluent in English, but many of them are first-generation English speakers and I'm pretty sure English was the second or even third language for many. All that is to say, I've been on that other side of the coin, so for me to completely miss the point of what was going on, while it was happening, was a real wake up call. But I'm almost glad that I missed it at first, because it allowed me to feel real emotions that—while only the tip of the iceberg—an ESL student would face.

I'm so grateful we had that experience because if I'm ever in that type of scenario again, I will be hyper-cognizant of exactly how I am handling myself and communicating. Not to say that I wasn't empathetic to those dealing with language barriers before, because in all fairness I was, but it was a real (sorry, gotta bring in Oprah again) "aha! moment." I've done immersive French language programs in high school where I felt completely out of my depths, but that is such a different situation. You're there in a privileged situation to learn a language, not learn math or another subject, and the expectation is that you're in the same boat as your peers. I can now only fathom how isolating it must feel for some students to feel like they're the only ones struggling to understand a language, much less understand math concepts being taught in

> said foreign language. Anyways, I wanted to get the chance to flesh out my thoughts on this because it was truly one of the most humbling, effective, and thoughtful educational experiences I've ever had, and I'm really thankful for the opportunity to have that. It'll stick with me for my entire teaching career, I know! —Caitlin

Erica and Caitlin both give us much to ponder about how we math educators can provide "original source" experience that shakes up the status quo especially for individuals who have never quite felt totally out of their league, so to speak. This touchstone strategy has worked in my classes, and I recommend it wholly.

Touchstone Strategy #8: Concept Teaching Games

Have each student plan and share their own concept teaching game.

When I started teaching Math for Teachers at Bank Street in 1974, I was committed to asking my students to design concept teaching games. During my years of teaching elementary school students and with exposure to the Madison Project math (Davis, 1984), I had used games as a medium to deeply teach the conceptual underpinnings of each big math idea. I also aimed to decenter myself from teaching so I could watch kids in process and assess their knowledge. The "concept teaching game" tool allowed me to do that. By carefully observing students at play, I could truly understand what they understood in math and what they struggled with. With that information, I could tailor plans for follow up. The games gave me the assessment information I needed, and I wanted to help other teachers learn to do that.

With that background, I decided to require the same of each of my students attending Math for Teachers. For the past 43 years, I have required the design of a concept teaching game. At the end of the process, my students' "write ups" and photos were placed in binders so other members of the class could access each game, including a description of what math concept the game taught and how to play. In later years, lesson plans for implementation were added as a requirement. All told, until the advent of Google Docs, I had about 35 years of game binders in our Bank Street Math Resource Room.

In this assignment, I always aim to carefully distinguish between concept teaching games and skill reinforcement games. They are totally different animals. For games to be classified as "concept teaching," you need to insure that the learner who has never before seen the concept, can, in fact, take ownership of the concept by playing (e.g., learn that multiplication is repeated addition or the inverse of division or can be represented using rectangular arrays, etc.). By contrast, a skill reinforcement game simply takes already learned concepts or skills and asks kids to practice them as known facts (e.g., 8×5 does equal 40). A skill reinforcement or practice game might ask the player to pick a card like 8×5, answer it, and move one space on a board. Nothing in the game invites the learner to see why $8 \times 5 = 40$. They do not see an array with grid paper; they do not have to collect 5 towers of 8 cubes or 8 towers of 5 cubes, or even use a hundreds board to skip count by 5s or by 8s the required amount to get to 40. Such a skill reinforcement game does nothing to insure conceptual understanding of what 8×5 looks like or the multiple ways you can deconstruct 8×5 [as $(5 \times 5) + (3 \times 5)$, for example].

Leah Silver, my teaching assistant on and off for the past two years, developed additional tools to help our students appreciate the continuum of concrete to abstract thinking required by concept

teaching games. Students were asked to design games that focused in the concrete/pictorial realms with only advanced levels moving to the abstract realm.

I have come to expect that people would resist and balk at being asked to make their own concept teaching game. And a few did every semester. Even some faculty told me they felt the concept teaching game assignment was burdensome, anxiety provoking, and not necessary, given all the fine new curriculum materials people now are using in math classes in elementary schools (e.g., Investigations in Number Data and Space, Bridges, Math in Focus, Envisions, etc.). I resisted their resistance because of what I was reading in my students' journals each and every semester.

Students often struggled to decide what a concept was, struggled to select one, were challenged to think about how they could teach something brand new to children, and struggled with design elements and how to engage children. I say, that is good! Their job as new teachers is to learn all that.

The frustration these adult students share with me and my assistants give us additional insights into who they are as teachers, what math knowledge they bring to this problem, how well they take our advice, how skillfully they adapt our advice, and how true they are to the purpose and constraints of the assignment. The emotional pleas and draft ideas they present to us in journals provide me the missing link for me as their teacher. As a course instructor, I rarely if ever see my students teaching. But I can read their thinking about "teaching" when they write me with nascent ideas regarding picking a concept (not just a skill), planning how to teach a new concept, planning how to keep kids engaged, and planning how to use the idea to assess learning. This is as close as I can get as a course instructor to seeing what goes on inside a teacher's mind when developing plans to enact teaching.

Every week I would model a well-crafted concept teaching game and its rules so they could taste a wide range of them all semester. Some that we always used were: Action Fractions, The Joining Board Game, How Big, Make Ten, Build a 100, Racing Dice, Capture Five, The Factor Game, Target 1000, and Make Five. Slowly they become more and more aware of what a concept teaching game could look like.

Because I want always to model teachers working collaboratively as a community of learners, only six people per session are asked to share their progress in designing a game. The other 20 or so teachers were free to roam around the room playing each game and could offer suggestions for clarification and perhaps improvements. No teacher should ever feel alone in developing teaching methods or ideas.

My underlying goal in this assignment is to assess if an adult student is capable of designing a complete math experience that focuses on a learner-centered activity and gives ownership of the math to the student. I know this is a challenge, but why is a teacher attending Math for Teachers if not to learn to do that? As you will see from the two journal comments below, emblematic of many entries I have read, the concept teaching game assignment is challenging at first. I believe this challenge, once met, is usually felt to be very worthwhile and I recommend this activity be implemented in courses for new teachers. Once again, I learned about the students' tenacity for sticking with an idea for a semester and revising and improving it based on feedback from both the instructors and fellow students. Though other faculty might not agree with me, I feel this was one of the best assignments I could have given my students. I learned so much about them and I had fun with their creativity as well. I cannot tell you how many of our students' games came with me to districts across the US and to teachers in other countries as well.

Figure 4. Sharing a concept teaching game

On making a game:

> I actually love the game-making experience much more than I thought I would. I had a mental impediment with it. I think part of it I can attribute to the fact that I just don't have experience creating lesson plans or games, and it was all sort of new and uncharted territory. I had a couple false starts with creating the game—going down one direction and then abandoning it altogether. But it was such a helpful and clarifying experience. My classmates—and Hal!—had such great ideas and constructive criticism about the various directions my game can ultimately go in. I thought that I would treat this game as purely a class assignment, and not necessarily something I would take with me, but I couldn't have been more wrong. I'm going to continue tinkering with this game through the years, and ultimately I would like to develop this into a "chutes and ladders" style game, like Hal suggested. How I'll execute that is a bit of a work in progress, but I'm thinking on it and am genuinely excited to have a polished product to take with me into classrooms. And I can even do different iterations of the game. But it was a huge mental hurdle that I got over, and I'm genuinely excited to create more games in the future. —Caitlin

On overcoming fears and meeting goals:

> To address the very first point, about taking risks: I think I've progressed in this area. The piece of this course that helped me most with this was the concept teaching game. It was the assignment I was most afraid of—spearheading a whole game all by myself and then teaching it all by myself to classmates that I've seen excel all semester. I was terrified. Fortunately for me, the scheduling put me at the very beginning which meant I needed to jump in with both feet. I asked a classmate if she might play the game with me beforehand and she agreed! That was a risk because I let someone give me feedback, though it was also a safety net because it helped me preview the work

> in my preferred modality (one-on-one). This reminded me how some of my students may need the chance to preview a project in a different (possibly smaller) setting before presenting it to the class as a whole.
>
> In presenting it to small groups, I was impressed with the feedback I received. I was also surprised how well I was able to hear my classmates. I thought I was going to show the game, explain it, and then everyone would tell me that I was unprepared and the game was stupid. This did not happen. I got some feedback that helped me develop my core idea and also inspired me to change a few simple elements to make the game more accessible. I was able to move my Ego out of the way and listen to the feedback as it was coming to me.
>
> I found this experience to be helpful to both of my goals: I was able to successfully take a risk and I also learned some ways to support different kinds of learners. I noticed that I needed some extra one-on-one time, which could be something I could pass on to math students in the future. I also noticed that my classmates had some trouble with the organization of the game, which showed me clearer ways to display information for all types of learners. Yay! —Holland

As each semester came to an end, I grew more and more excited to see what the final four sessions would bring. For the 30 to 40 minutes of each two-hour session, students had signed up to share their games at each of six tables in the room (no more than six games each week). The balance of the class would rotate and play each game for just a few minutes. It was like going to a game fair for the last four weeks of the semester. Everyone would get a sense of the purpose of the game (the concept it teaches . . . or fails to teach) and how to play. Then they were asked to give hot and cold responses to the game maker. Here again I put each teacher into the position of being a professional supporter of a colleague, sort of a critical friend who can give substantive teaching support to one another. Another goal was to create and model what a supportive educational community looks and feels like. I aimed to provide emotional support for the community of learners. As my assistant and I roamed the room, we not only saw the evolved games being played, but we also watched and listened to the tone and content of the support that other students are able (or not able) to provide. Once again, the emotional tone in the room is what I am learning from. That gives me the opportunity to observe, follow up with each teacher, and provide thoughtful productive commentary on each person's teaching, conceptual understanding, and capacity as a constructive colleague.

See samples of concept teaching games that were shared in Figure 5 (next page).

Figure 5. Sample concept teaching games

Conclusion

Bringing joy to uninspired teachers of math.

My dream has been for a generational shift in math teaching spearheaded by the teachers who graduated from the Teaching and Learning Department at Bank Street. In my work, I have intentionally linked a teacher's emotional life to their thought process during a math teaching re-education process. One's feelings or affect and one's intellectual engagement to study are always intertwined (Brakett, 2017). In mathematics, I have found that negative feelings about the subject often overpower the intellect. If the feelings towards math are negative and are not challenged and remediated, the potential effect on our graduates' classes of children will remain bleak.

Every one of our adult students had already been through grades pre-K - 6 math prior to attending our class. So the math we teach in Math for Teachers is not new. It may, however, be delivered with a very different focus. I advocate both an inspired learning process as well as an unlearning/re-learning process. To inspire other teacher educators, I hope I have built a case for addressing emotional impediments and negative triggers in math learning and the reasons to provide positive replacements for those feelings for teachers. Teachers for years have told me that their perceptions of themselves as mathematical thinkers change as a result of attendance in Math for Teachers (Melnick, 1992).

In recent classes, due to my longevity at the College, I have had teachers announce on day one that their mothers took classes with me in the 1980s and, to this day, recall how powerful their relearning of math was. The residual learnings from Math for Teachers can be outstanding. I feel it is largely emotional. I also believe that most teacher educators in the country can do this if they take care to value the emotional learning experience equally with the cognitive work. Learning math in the way we have come to teach it at Bank Street can change the lives of future adults in our society. If we sensitively plan for change and remain excited about the prospect, change will happen.

A graduate school teacher educator usually can only assess what his students learn by observing them in the graduate classroom or by requesting papers written during the instructional semester. Course instructors rarely have the time or the luxury of visiting each student in their field placement. I know I often wonder what has been learned in the long run. Did anything that I taught stick? Sometimes we find out.

In the last paragraphs of this paper I share with you a description by a student who attended my class seven years ago in 2011. Tyler Jennings took one course with me to test out Bank Street as a potential place to do his graduate studies. He was, at the time, an experienced fourth grade teacher at Caedmon School on the Upper East Side of Manhattan. While at a social gathering this past month, I happened to overhear Tyler describing to others how, from the time he was a child, he had always disliked mathematics until attending my Math For Teachers course at Bank Street. He described that experience as transformative and said it totally changed not only his relationship with mathematics, but also his career trajectory. I close with his (and one other) story because both capture nicely how teacher education must address the emotional as well as the cognitive if it is to be effective. You need to teach "affectively" to be effective.

Tyler explains what he calls "Tyler's Math Journey" from a math phobe to a math coach, working as a professional developer with teachers today at his school on the Upper East Side of

Manhattan. Listen for the emotional, address the intellectual, help teachers connect the two. Help them appreciate mathematics for what it always has been —simply the study of relationships and the science of pattern. Help them to feel it and then leave them to inspire others.

Here is Tyler's story:

> Until the age of 25, I believed that I was not made for math. As far back as I can remember, certainly third grade, I knew with conviction that I was a poor math student and that I disliked the subject. I noticed the way that adults spoke differently about me when the subject of my math learning arose; the tone in their voices changed, became pinched and awkward rather than proud. Sometimes I would imagine an expert who could enlighten me: "It's really quite simple," they would say, "you don't have a math mind, but you can be good at other things." I ceased to pursue my math studies as soon as possible. If a high school math course was not mandatory, I did not take it. I chose a university without core requirements, so that I could avoid college math altogether.
>
> Today, I work as a Math Coach at a pre-K and elementary Montessori School in New York City. Since I also serve as the school's Director of Curriculum and Innovation, I work with teachers in other subject areas, too, but I know that I behave differently when I approach math curriculum and instruction: I come alive. I feel that I am at my most passionate and engaged, successful and talented, helpful to teachers and impactful in children's lives. I conduct far more professional research into math content than any other subject—I can't get enough of it!
>
> So what happened? At the age of 25, when I enrolled at the Bank Street College of Education, and specifically in a course called Math for Teachers, a dangerous cycle was disrupted. I was invited: 1) to share my math journey, including its emotional dimensions, and 2) to experience math differently. I was welcomed in; it was made immediately clear that, at Bank Street, there would be no outsiders to the pursuit of math learning—that math learning could be for all, could be for me. And we did math together, which made it possible to redefine our very concept of what math is: an active pursuit of meaning-making, the study of the science of pattern. So much more than an inert body of knowledge to store (or fail to store) in the mind, which is, of course, the way that I was taught math. I cannot express how cathartic, emotional, and exciting that process was, and continues to be even as I write this.
>
> Without my transformative experiences in this teacher education program, I know that my own negative experiences as a young math student would have directly informed my choices as an educator. Not only would I have mimicked the way that I was taught, but I would have communicated, subliminally or not, my dislike of math to the students. I could have easily become the expert who told certain children that they simply did not have a math mind, but that they might be good at something else. It seems to me that an excellent teacher education program may be the best chance for disrupting this dangerous cycle for future educators. It certainly did for me. —Tyler

I wholeheartedly believe that the impetus for the transformation Tyler speaks of is Bank Street's longstanding commitment to balancing theory with practice. Instruction in all our math classes consciously aims to balance current, well researched theory equally with the emotional experience

each teacher brings to her studies. The catalyst that connects theory to practice, public knowledge building to personal, is deep reflection.

Let's hear from Caitlin one more time as she reflects on the course one full year after its completion. What is it that triggered her massive change? What was it that truly changed?

> I really remember benefitting from you asking us to dig deep in our own personal math experiences. My math experiences were happily filed under "The Past: Do Not Open." And, for lack of a better comparison much like therapy, it's by revisiting and delving into it that we are able to grow, heal, and move on. You brought joy back into the subject with in-class projects. I got so into the making my own concept teaching game in particular, it really returned ownership of the subject after all those years. —Caitlin, Spring 2017

This year, I have begun to visit local schools to meet with and observe teachers who attended Math for Teachers at Bank Street, and whom I have taught. In the New York City area, I have seen the next generation of math teachers from Bank Street flourishing. Math in these schools is taught with feeling by inspired teachers and with math-inspired children. In more than one of those schools, math has become the students' favorite subject and, in some cases, the teacher's favorite subject to teach.

What does that foretell? Math teaching can change for all people if we ensure that teachers see math in all its dimensions and recognize both the emotional as well as the intellectual.

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