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A Curriculum Review Checklist to Support Teaching for Robust Understanding

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Abstract. Part of the professional work of teaching includes selecting curricular materials. The selection might be small (for a lesson or a project), medium (for a course one is teaching), or large (for a school or district). Sometimes the selection is done by one person, sometimes a committee. Whatever the circumstances, research suggests that having a framework for examining materials and methods is valuable. The first half of this report describes how such a framework might be designed in alignment with the goals of the Teaching for Robust Understanding approach while also supporting responsive instruction. The second half of the report provides an example, in checklist format, that the reader might use immediately. The checklist consolidates the research-based experience of veteran teachers into a useful tool for a newer teacher.

Introduction

As teachers we apply a personal "theoretical framework" about what makes for good teaching in the decisions we make about instruction. But our framework may not be well-organized or even thoroughly examined. The checklists in this document, which can be used to help a teacher decide on the usefulness of a curriculum, are based on a framework we (the authors) have found useful called Teaching for Robust Understanding (TRU; Schoenfeld et al., 2016). The checklists offer a structure for considering all five of the dimensions defined in the TRU framework (see Figure 1). The checklist is a tool for ensuring that the content is appropriate, the cognitive demand is rigorous and supportive, that all students can access the learning opportunities, that students get to have a say in their learning and build their identity as learners, and that the teacher has direction for next steps after a given lesson. Using these checklists can help teachers select quality curricular pieces and improve implementation of existing curriculum; in addition, it can help to shape and clarify a teacher's own theoretical framework and provide a starting point for future professional learning. To use the checklists, it is important to first understand what the TRU framework dimensions are, what they look like in practice, and how they are related to improved student learning outcomes.

The checklists are also influenced by the principles of Complex Instruction (Cohen & Lotan, 2014). TRU and Complex Instruction both provide ways to unpack teaching, though in somewhat different ways. Complex Instruction focuses on the intersection of three key components: classroom norms and student roles, status interactions, and groupworthy tasks (Horn, 2012). Complex Instruction is primarily about how to establish and maintain positive relationships among and between all students, the teacher, and the mathematics itself, which is essential for equitable access to the curriculum. The TRU framework is more focused on what the

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outcomes should look like—that is, how to observe or evaluate the effectiveness of a teacher's practice. We believe TRU is also a useful tool for examining curriculum, particularly for identifying curricular supports for the kind of teaching promoted in Complex Instruction. Our intent in writing these checklists is to help you identify a curriculum strong enough, deep enough, and supportive enough to be a base for robust teaching and learning.

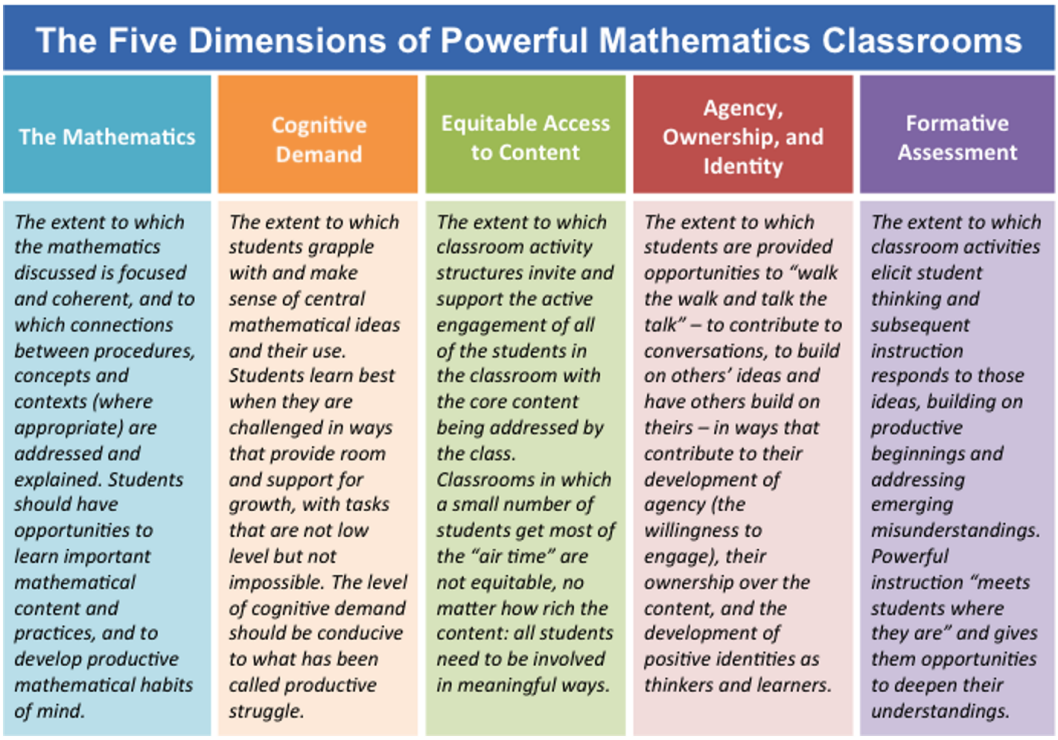

Figure 1. Summary of the TRU framework (Schoenfeld et al., 2016).

Position of the Authors

Before moving to the intentions around and uses of the checklist, we offer short professional biographies. These allow the reader insight into the people behind the checklist offered here.

Daniel. When I first started teaching Algebra, I tried incorporating problem-solving in addition to the required procedural work. But this was the beginning of NCLB, and each year, the STAR tests continued to grow in importance. My school was laser focused on STAR scores, so my teaching became more and more procedural and targeted to the released questions. Problem-solving and anything else that was interesting disappeared. Up to this point, I had been teaching with an emergency credential, but by my 5th year of teaching, I decided that I really did want to be a teacher. I enrolled at San Jose State University, and got my credential in the evenings, as I taught full-time on an intern credential (which means I never really had a mentor teacher). The program's overall quality was mixed, but I had a very good experience with my mathematics methods teacher, Ferdie Rivera. From him, I learned about the socially constructed nature of mathematics learning, and I learned a great deal about how to actually structure learning experiences for my students. My teaching began to improve again, even under the oppressive environment caused by NCLB. But by my 10th year of teaching, I felt like I had

plateaued, and I was still not getting my students to learn or be engaged in the ways I believed they could. I was building strong relationships with them, but not helping them build strong relationships with math. I was starting to feel like my choice to teach mathematics had been a mistake, and that I could not keep doing it for much longer. The adoption of the Common Core turned things around for me, because it finally allowed us to teach thinking, communication, and problem-solving skills. The transition to Common Core, however, was a tough one for many teachers. And even now, 10 years later, many mathematics teachers are still teaching procedures and concepts, and not putting the mathematical practices at the center of learning. I eventually became the co-chair of the mathematics department at my school site, and began encouraging people to use new instructional practices that focused on student discourse and problem-solving (math talks, notice and wonder, MARS tasks, 5 Practices, Routines for Reasoning, modeling tasks, and so on). I read tons of books, blogs, and twitter feeds and really increased my own mathematics teaching knowledge. I am currently finishing up my masters in mathematics education at San Francisco State, where I've learned the value of teaching heterogeneous groups of students with the tools of Complex Instruction. I am also the mathematics coordinator for my district, and I am working with teachers to bring these practices into as many classrooms as possible, using the TRU framework as a guide.

Rebecca. In my first year of teaching, I struggled a lot with classroom management with a group of students who challenged me. The second year, I felt more confident. I had gone through the curriculum before and I had only one period of Algebra 1, which allowed more flexibility of pacing. I remember really being able to emphasize conceptual understanding using multiple representations. Students were able to take their time making their tables, graphs, rules, and tile patterns to show how the growth and starting amount were connected across each representation. While I know there were many factors at play, I felt like I was seeing my own success in the students' progress, especially as they continued to be successful in mathematics classes the following years. This year, my eighth as a teacher, I saw another big jump of progress in my teaching. Last spring through this year I've been learning more strategies and ideas through classes, workshops, conferences, and collaboration. Most notably, I've learned about and begun to implement Complex Instruction. The structure of Complex Instruction has made a profound difference in my teaching and my students' learning. It has fixed problems that I didn't even realize were issues until seeing the progress my students have made. I first bought in to using Complex Instruction because it supported my students in group work. However, in learning about and being able to address status issues, I've been able to see formerly low-status students really shine. This is particularly notable in some students who I had in previous years who had seemed to struggle despite my efforts to help them; now with implementing Complex Instruction, those same students are respected and have their classmates truly valuing their ideas and contributions both during group work and in whole-class discussions.

Intentions

Choosing, designing, and implementing curriculum effectively is a large and complex task. Of course, the accuracy of the mathematical content is essential. Beyond that, there are so many different things to consider that, even for a veteran teacher, it is easy to overlook crucial components. Deep examination of the TRU dimensions will provide a reflective teacher with the opportunity to make significant improvements to their practice. The original TRU documents, however, are intended to be a lens into examining a teacher's enacted practice (as they consist of observation and conversation guides). We wanted to develop a TRU-based tool that would be

useful for analyzing and selecting curricula, especially for teachers still in their first years of practice.

While TRU provides a comprehensive way of thinking about mathematics instruction, there are certain areas that would benefit from more explicit attention, such as supports for language learners and authentic connections to students' lives and experiences. We therefore added statements about these components within the appropriate dimensions. We relied on the Culturally Responsive Mathematics Teaching (CRMT) framework (Aguirre et al., 2012) for ideas in these areas.

In designing this tool, we include a Goal (on the left) that embodies a curricular need and Checkpoints (on the right) that are statements about how the curriculum can address the need. A rapid first-pass could darken (or not) the checkboxes—as we have done in the example in the Appendix. A more careful pass might include partially filling a box and making notes on what is missing from the curriculum. The checklists should not be considered comprehensive, but rather a jumping-off point for supporting teachers as they consider each idea. We hope that using this tool will help teachers make more informed choices about the curriculum that they implement in their classrooms, facilitate professional learning, and improve the quality of mathematics instruction that students receive.

The Checklists and Their Use

From using these checklist tools, we hope that teachers will be able to get a quick sense of the overall quality of the lesson they are evaluating, and how well it aligns with their intentions. This tool can help teachers sort lessons into one of three categories: can be implemented as the curriculum suggests, needs minor adjustments before implementation, or needs a significant amount of work before implementation. If comparing curricula to make purchasing decisions, we recommend using the checklist on all lessons of at least one full unit per curriculum to get a sense of each in order to effectively compare. If using the checklist to make decisions about materials, examine enough lessons to provide a comparison or synthesis of information across lessons. If using this to compare and choose the best lesson for your needs, then use the checklist to see which lesson is the most ready-to-implement. If using this on a selected curriculum with no alternate options, use this checklist to see what areas need modifications to be improved.

The reader may notice that there is some repetition across the goals in the different dimensions. The five TRU dimensions are, of course, highly interrelated, so it can be difficult to fully separate them. As teachers may choose to focus their curriculum evaluation on only one or two of the dimensions at a time, we tried to make each checklist independently useful, leading to some repetition. As teachers practice evaluating curriculum using these checklists, they will get better at seeing all of the components of effective lessons that span the five categories. We hope more teachers will become aware of and prioritize having all students (1) discuss and share their ideas as well as (2) use students' own cultural backgrounds to enhance understanding. These are the two areas that we found repeatedly lacking in lessons across different curricula, though these areas have a substantial impact on student learning. With more teachers learning to include student talk and cultural relevance in lessons, we hope to see more options to include these pieces in the official curriculum. After presenting the checklists on the next few pages, we return to this idea and offer some questions and strategies that address implementation, once a curriculum is reviewed.

Mathematics

Goal	Formal Curriculum Checkpoints
It is clear how the mathematical ideas for this course develop in this lesson.	❑ Alignments with standards are stated ❑ Connections to standards are explained to teacher ❑ Connections to standards are made explicit to students
Learning intentions and corresponding success criteria are clearly established (for **math content and practices**; see Hattie, 2012).	❑ Clear indication of the lesson's key math content (i.e. concepts and procedures) ❑ Connections to math **content standards** are explicitly stated ❑ Connections to math **practice standards** are explicitly stated ❑ Success criteria are presented in a student-friendly way (i.e. similarly to how they would be found on a rubric)
Lesson connects with what students already know.	❑ Requisite prior knowledge is clearly indicated ❑ Lesson explicitly builds on students' prior knowledge. ❍ with new modification ❍ as an extension ❍ by connecting multiple ideas
Lesson notes indicate how student difficulties with prior concepts or procedures may emerge.	❑ Potential problematic areas are detailed in teacher notes ❑ Suggested modifications are provided
Students interpret, create, problem-solve, or communicate with multiple representations of mathematical ideas.	Students are expected to use the following representations or to make explicit connections between representations: ❑ Physical (manipulatives, realia) ❑ Visual (graphs, patterns, diagrams, video, images) ❑ Symbolic (expressions, equations, notation) ❑ Verbal (using math vocabulary) ❑ Contextual (real-world situations)
Students engage in mathematical proof and validation.	❑ Students work on aspects of proof and argumentation, such as: ❍ Examples vs. counterexamples ❍ If-then statements ❍ Always/sometimes/never statements ❍ Conjecturing and testing ❑ Students are asked to ascertain whether an idea is true ❑ Students are asked to convince others that an idea is true ❑ Students work on formal mathematical proof (logico- deductive, inductive, proof by contradiction, etc.)

Cognitive Demand

Goal	Curriculum Checkpoints
Students have opportunities to make sense of mathematical content.	❑ Problem-based/inquiry-based lesson ❑ Students discuss emerging ideas in small groups or as a class ❑ Students engage in written reflection about emerging ideas ❑ Productive struggle is planned, with appropriate supports
Students have opportunities to engage in mathematical practices.	❑ Students work on non-routine problems that have multiple solution pathways ❑ Students must reason about quantities and relationships ❑ Students build, communicate, and critique arguments about mathematical ideas ❑ Students engage in mathematical modeling ❑ Attention is given to specific mathematical tools that are appropriate for the given task ❑ Explicit attention is given to precision ❑ Students reason about the structure of problems, connections between mathematical representations, or compare/contrast with previously learned ideas ❑ Students use patterns to make sense of a problem, or to generalize
According to pacing information, students are given an appropriate amount of time for each task.	❑ An appropriate amount of time is allocated for each task ❑ New times are specified if including any modifications suggested by the lesson notes Note: Too little time may cause students to feel pressured or rushed to come to a conclusion. Too much time may lead to unproductive struggle or loss of focus.
There is an appropriate level of cognitive demand in lesson activities.	❑ Students spend time working on tasks of all types of cognitive demand (variety) ❑ Students spend sufficient time working on higher-level demand tasks (challenge) Note: For more on types of cognitive demand (e.g., *memorization*, *procedures without connection*, *procedures with connections*, *doing math*) see Smith & Stein (1998).
Students are supported in sense-making, yet the intended level of cognitive demand has been maintained.	Lesson notes provide: ❑ anticipated misconceptions and ways to address them ❑ differentiated learning supports, in general ❑ specific supports for language learners and students with special needs ❑ participation structures for small-group work Lesson utilizes: ❑ non-routine problems ❑ low-floor, high-ceiling problems (everyone can access the problem and there are built-in opportunities for extension)

(Cognitive Demand checklist continued on the next page)

Cognitive Demand (continued)

Goal	**Curriculum Checkpoints**
Students are given a chance to explain things (not just provide answers).	❑ Students are expected to explain their ideas or processes: ❍ in writing ❍ in pairs or small groups ❍ to the whole class ❑ Students are expected to justify or defend their reasoning ❑ Students are expected to critique the reasoning of others ❑ Students are expected to revise their explanations based on feedback
Students are held accountable for high-level products and processes.	❑ Students engage in significant written explanation/reflection about their work ❑ Students present their work to the class or in small groups ❑ Groupwork tasks include both an individual and a group product that can be assessed

Note: Equity and Access Checklist begins on the next page.

Equity and Access

Goal	Curriculum Checkpoints
All students get to participate in mathematics learning in meaningful ways. Students cannot hide or be ignored.	❑ Lesson activities provide students an *opportunity* to participate meaningfully ❑ Lesson activities make it hard for a student to opt out of meaningful learning ❑ Lesson activities *require* all students to participate meaningfully (for example, groupworthy tasks where the group can only succeed if all members of the group are engaged)
Students are kept engaged with the material.	Lesson includes: ❑ Content that connects to real-life problems or experiences (i.e. mathematical modeling prompts) ❑ Movement ❑ Games / puzzles / challenges ❑ Problem-solving ❑ Communication activities ❑ Reflection activities ❑ Meaningful student choice
The lesson supports students' academic language development, and draws on their linguistic funds of knowledge (Aguirre & Zavala, 2013)	❑ Text includes explicit supports for language learners, such as sentence frames or Math Language Routines (Zwiers et al., 2017) ❑ Lesson explicitly asks students to contribute language from their personal experiences, such as words/forms of communication from: ❍ other classes (e.g., science, English) ❍ outside of school experience ❍ home language(s)
Learning intentions and corresponding success criteria are clearly established (for language and social skills development).	❑ Key general academic and domain-specific vocabulary words are specified (e.g., Tier 2 and Tier 3; Beck, 2002) ❑ The lesson explicitly describes and supports norms for: ❍ Cooperation ❍ Communication ❍ Active listening ❍ Giving and receiving help ❍ Building on others' ideas ❑ Language and social skills development have clearly stated learning intentions both in teacher notes and student-facing text
Lesson activities are structured to promote equity of voice and participation.	❑ Students have individual think/work time before being asked to speak ❑ Students are expected to build on each other's ideas ❑ The text supports the use of the five practices for orchestrating classroom discussions (Smith & Stein, 2018; Smith et al., 2009). ❑ Lesson notes provide structures to support equity in both small group and whole class discussions
Lesson activities are designed in a way to mitigate/disrupt status issues in the classroom.	Lesson activities provide opportunities for students to: ❑ Demonstrate mathematical competence in multiple ways ❑ Collaborate rather than compete ❑ See the value in examining others' ways of sense-making ❑ Celebrate the work of others

Agency, Identity, and Ownership

Goal	Curriculum Checkpoints
Students have opportunities to generate and share their own ideas. Students' ideas are built upon.	❑ Includes questions with no single "right" answer ❑ Includes directions to "generate", "justify", or "show how you know" ❑ Students are asked to convince or persuade others of their own ideas ❑ Students present work to the class ❑ Students are asked to apply a classmate's strategy to a new problem/situation
Students have opportunities to construct new understandings of their identities as math-doers.	❑ New and different ways of thinking/doing math are highlighted by the text ❑ Students are asked to reflect on connections made and what they contributed to the lesson
Students are recognized as being capable and able to contribute.	❑ Lesson activities allow for student choice ❑ Lesson activities are not over-scaffolded ❑ All students are expected to contribute ❑ Students expected to share more than just computational answers
This lesson helps students connect mathematics with relevant/authentic situations in their lives.	Students are asked to: ❑ Provide examples from their experiences that illustrate mathematical concepts ❑ Research related situations ❑ Pose/solve new problems based on personal experiences ❑ Engage in modeling to solve real-world problems ❑ Use math to understand/critique/change an important equity or social justice issue
Students are expected to assess themselves in this lesson.	❑ Written reflection ❑ Self-assigned score on a rubric ❑ Peer assess/document what was contributed by group members ❑ Focus on growth over time

Formative Assessment

Goal	Curriculum Checkpoints
Formative assessment strategies are present in the lesson.	❑ Specific strategies, like exit tickets or use of individual whiteboards, are provided by the text ❑ Teachers are told what to look for in partner/small group activities, such as card sorts, and suggestions for responding ❑ Technology tools that can be used to check for understanding are included ❑ Teacher text indicates likely misconceptions and provides suggestions for how to respond
Student ideas, including non-standard conceptions, are used to inform subsequent parts of the lesson.	❑ Ideas are used to build future strategies ❑ Ideas are discussed to highlight pros and cons of different strategies ❑ Non-standard conceptions are highlighted for further discussion/investigation ❑ Problematic conceptions are turned into "find the error" problems ❑ Lesson activities branch, depending on outcome of prior activities
Text provides a way to assess students' work in collaborative pairs/groups as well as their emerging mathematical ideas.	❑ Rubrics, checklists, or reflective activities connected with collaborative work are included

Conclusion

As experienced teachers, we tend to jump into curriculum analysis by heading for the first meaty mathematics. To illustrate the use of the Equity and Access checklist, we took a look at Unit 2, Lesson 1 from the Grade 7 *Illustrative Mathematics* curriculum. In the Appendix we include our version of a completed checklist for the Equity and Access dimension for the lesson. What we noticed included the ways that "hands on" experiences for students were called for in lesson implementation. More generally, once you have identified a reasonable curriculum, there are some questions that are important to address to prepare for an implementation that is equitable and responsive to students.

We close with important overarching questions related to the three TRU dimensions that may be the least familiar to new teachers: equity and access; agency, identity, and ownership; and formative assessment. These dimensions focus on the nuances of implementation required for establishing, maintaining, and assessing the effectiveness of instruction for every student. These questions are a purposeful enrichment of the TRU framework with ideas from Aguirre and Zavala's (2013) lesson analysis tool for making responsive teaching explicit.

Equity and Access: Implementation Questions and Strategies

- How do you know you established norms in your classroom that will support students' ability to work productively at the desired level of cognitive demand in the lesson?
- How will you make sure when teaching with worthwhile, challenging, and engaging problems/materials that you do not unintentionally lower the intended cognitive demand (e.g., by over-scaffolding lesson content)?
- Have you considered specific scaffolds of student participation? How do they provide all students, individually and collectively, access at the intended level of cognitive demand?

Strategies to consider in answering these questions:

- Learning about Complex Instruction is useful (e.g., complexinstruction.stanford.edu).
- From Complex Instruction come tools for teaching that puts students in pairs or small groups to support individual understanding, including defined roles and responsibilities, providing language supports for students, and using both individual and group accountability measures.
- Attuning yourself and students to status issues in the classroom and learning to intentionally disrupt them is of utmost importance. Too often, group work fails because of insufficient use of participation structures, lack of plans for student accountability, and status differentials among students.

Agency, Identity, and Ownership: Implementation Questions and Strategies
Classroom discussions are an integral part of robust teaching, but there are some important things to consider to ensure successful implementation.

- How will you decide who is selected to talk during the whole class discussion?
- How can you ensure that a variety of student strategies will be surfaced, and not just those who are more vocal?
- What steps can you take to interrupt status problems as they emerge, and to head them off by making instructional choices?

Strategies/resources to consider:

- During work time, find students with several different strategies to have them share out
- Pay attention to good strategies from lower status students and have them share their ideas (be sure to reinforce the good thinking to assign competence)
- After partner/small group work, use equity cards or sticks to call on students randomly (not taking volunteers)
- After group work, have one person share out from each group (students should know how share out will work: student can be chosen randomly or use process to select as "reporter")

In addition to getting students to participate equitably, they also have to be taught how to listen to and value each other's contributions.

- What strategies will you use to ensure that students actively listen to each other and seek to build on each other's ideas?

Strategies/resources to consider:

- Students work together on strengthening each other's ideas
- Students provide each other with feedback to make revisions
- Student partner talk routines that include responding each other (beyond announcing ideas to each other, *responding* to what the other person has said)
- Teacher facilitates students repeating what they heard before building on a previous person's idea

When student discourse is a large part of a lesson, you have to be prepared for students to take things in unanticipated directions. Are there some points in the lesson where students' emerging ideas have a significant potential to take the class in an unanticipated mathematical direction? What are your decision-making routines for when you follow their lead in the moment, and when you save their ideas for another time?

Formative Assessment: Implementation Questions and Strategies Related to the last point above, mindful planning of formative assessment can give you the instructional think-time you may need to process what is happening in the moment and decide on diverging from your intended direction?

- How will you respond? What will be a few "go to" formative activities?
- Does instruction respond to students' ideas and help them think more deeply?
- How will you give feedback to students on their work, in a way that is timely, meaningful, and yet manageable?
- How will students be expected to act upon the feedback that they are given?

Strategies to consider:

- Follow-up questions for students to provide explanations
- Partners critique each other's reasoning
- "Gots" and "needs" poll where each student writes one thing they now know/think based on what they are learning and one thing they are confused by or need to understand (better).

Author-Recommended Resources for More Information

For more information on some of the ideas referenced in the checklist, we encourage you to seek out these resources on the following topics:

(1) *Math Language Routines* from the SCALE project at Stanford (Zwiers et al., 2017)
(2) Complex Instruction:
    - *Strength in Numbers* book (Horn, 2012)
    - *Designing Groupwork* book (Cohen & Lotan, 2014)
    - SFUSD Website
(3) An overview that introduces many valuable ideas, definitely a good read for newer teachers: *Visible Learning for Mathematics, Grades K-12*.
(4) Smith and Stein's guidebook: *Book Preview: 5 Practices for Orchestrating Productive Mathematical Discussions*, see References, below, for related articles.

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Smith, M., Hughes, E., Engle, R., & Stein, M. (2009). Orchestrating discussions. *Mathematics Teaching in the Middle School*, *14*(9), 548–556.

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Zwiers, J., Dieckmann, J., Rutherford-Quach, S., Daro, V., Skarin, R., Weiss, S., & Malamut, J. (2017). *Principles for the design of mathematics curricula: Promoting language and content development*. Retrieved from http://ell.stanford.edu/content/mathematics-resources-additional-resources (Retrieved from Stanford University, UL/SCALE website.)

Appendix

1.2: Mystery Mixtures (15 minutes)

CCSS Standards

Building On

6.RP.A

Building Towards

7.RP.A

Routines and Materials

Instructional Routines

MLR2: Collect and Display

MLR7: Compare and Connect

MLR8: Discussion Supports

Required Materials

Colored pencils

Drink mix

Measuring cup

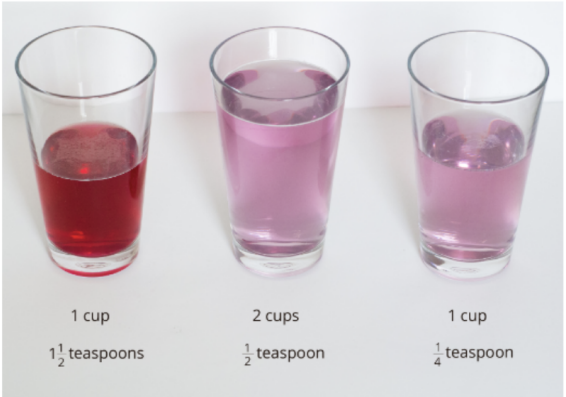
Measuring spoons

Mixing containers

Small disposable cups

Water

Activity	The purpose of this activity is for students to articulate that the taste of the mixture depends both on the amount of water and the amount of drink mix used to make the mixture. Ideally, students come into the class knowing how to draw and use diagrams or tables of equivalent ratios to analyze contexts like the one in the task. If the diagnostic assessment suggests that some students can and some students can't, make strategic pairings of students for this task.
Launch	Show students images of the drinks.



If possible, give each student three cups containing the drink mixtures.

Tell students to work through the first question and pause for a discussion. Ask questions like,

- "What does it mean to say that it has more drink mix in it?"
- "Imagine you take different amounts of the two that taste the same. There will be more drink mix in the larger amount, but it will not taste different. Why is that?"

The goal is to see that in the same quantity of each mixture (say a teaspoon), the more flavored drink mixture has more drink mix for the same amount of water. (Alternatively, we can say the more flavored drink mixture has less water for the same amount of drink mix.) Use MLR 8 (Discussion Supports) by making gestures or acting out facial expressions for "strength" of the mixture.

After the students have made some progress understanding this idea, the class should continue to the second question. If students finish quickly, press them to find the amount of drink mix per cup of water in each recipe, thus emphasizing the unit rate.

Access for English Learners

Conversing, Writing: MLR2 Collect and Display. Before students begin writing a response to the first question, invite them to discuss their thinking with a partner. Listen for vocabulary and phrases students use to describe how the amount of water and the amount of drink mix affects the taste of the mixture. Collect and display words and phrases such as "more drink mix," "more water," "tastes stronger/weaker," etc., and then encourage students to use this language in their written responses, and during discussion.

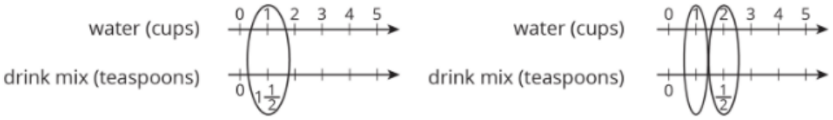
Student Facing	Your teacher will show you three mixtures. Two taste the same, and one is different. 1. Which mixture tastes different? Describe how it is different. 2. Here are the recipes that were used to make the three mixtures: • 1 cup of water with $1\frac{1}{2}$ teaspoons of powdered drink mix • 2 cups of water with $\frac{1}{2}$ teaspoon of powdered drink mix • 1 cup of water with $\frac{1}{4}$ teaspoon of powdered drink mix Which of these recipes is for the stronger tasting mixture? Explain how you know.
Student Response	Teachers with a valid work email address can click here to register or sign in for free access to Student Response.
Student Facing	**Are you ready for more?** Salt and sugar give two distinctly different tastes, one salty and the other sweet. In a mixture of salt and sugar, it is possible for the mixture to be salty, sweet or both. Will any of these mixtures taste exactly the same? • Mixture A: 2 cups water, 4 teaspoons salt, 0.25 cup sugar • Mixture B: 1.5 cups water, 3 teaspoons salt, 0.2 cup sugar • Mixture C: 1 cup water, 2 teaspoons salt, 0.125 cup sugar
Student Response	Teachers with a valid work email address can click here to register or sign in for free access to Extension Student Response.

Activity Synthesis

The key takeaway from this activity is that the flavor depends on both how much drink mix *and* how much water there is in the mixture. For a given amount of water, the more drink mix you add, the stronger the mixture tastes. Likewise, for a given amount of drink mix, the more water you add, the weaker the mixture tastes. To compare the amount of flavor of two mixtures, when both the amounts of drink mix and the amounts of water are different in the two mixtures, we can write ratios equivalent to each situation so that we are comparing the amount of drink mix for the same amount of water or the amount of water for the same amount of drink mix. Computing a *unit rate* for each situation is a particular instance of this strategy. Make these ideas explicit if the students do not express them.

If students do not create them, draw discrete diagrams like this:

Or double number line diagrams like this:



For each mixture, identify correspondences between the discrete and number line diagrams, and between the diagrams and tables:

water (cups)	drink mix (teaspoons)
1	$1\frac{1}{2}$
2	3

water (cups)	drink mix (teaspoons)
2	$\frac{1}{2}$
1	$\frac{1}{4}$

Ask questions like, "On the double number line diagram we see the 1 to $1\frac{1}{2}$ relationship at the first tick mark. Where do we see that relationship in the double tape diagram? In the table?"

Use MLR 7 (Compare and Connect) for students to compare methods of how they knew which recipe was strongest. Who used multiplication? Who used division? Who used a unit rate of water per drink mix teaspoon? Who used a unit rate of drink mix per water cup?

Access for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to illustrate correspondences between the number line diagrams and ratio tables for each mixture.
Supports accessibility for: Visual-spatial processing

Equity and Access

Goal	Curriculum Checkpoints
All students get to participate in mathematics learning in meaningful ways. Students cannot hide or be ignored.	■ Lesson activities provide students an *opportunity* to participate meaningfully ■ Lesson activities make it hard for a student to opt out of meaningful learning ❑ Lesson activities *require* all students to participate meaningfully
Students are kept engaged with the material.	Lesson includes: ■ Content that connects to real-life problems or experiences (i.e. mathematical modeling prompts) ❑ Movement ❑ Games / puzzles / challenges ■ Problem-solving ❑ Communication activities ■ Reflection activities ❑ Meaningful student choice
The lesson supports students' academic language development, and draws on their linguistic funds of knowledge (Aguirre & Zavala, 2013)	■ Text includes explicit supports for language learners, such as sentence frames or Math Language Routines (Zwiers et al., 2017) ❑ Lesson explicitly asks students to contribute language from their personal experiences, such as words/forms of communication from: ❍ other classes (e.g., science, English) ❍ outside of school experience ❍ home language(s)
Learning intentions and corresponding success criteria are clearly established for language and social skills development.	■ Key general academic and domain-specific vocabulary words are specified (Beck, 2002) ❑ The lesson explicitly describes and supports norms for: ❍ Cooperation ❍ Communication ❍ Active listening ❍ Giving and receiving help ❍ Building on others' ideas ■ Language and social skills development have clearly stated learning intentions both in teacher notes and student-facing text
Lesson activities are structured to promote equity of voice and participation.	■ Students have individual think/work time before being asked to speak ❑ Students are expected to build on each other's ideas ❑ The text supports the use of the five practices for orchestrating classroom discussions (Smith & Stein, 2018; Smith et al., 2009). ❑ Lesson notes provide structures to support equity in both small group and whole class discussions
Lesson activities are designed in a way to mitigate/disrupt status issues in the classroom.	Lesson activities provide opportunities for students to: ■ Demonstrate mathematical competence in multiple ways ❑ Collaborate rather than compete ■ See the value in examining others' ways of sense-making ❑ Celebrate the work of others