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# *Research to Practice Sampler*

## Engaging Students in Cognitively Demanding Tasks

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Abstract. Research to practice samplers provide a short review of the research literature on a topic and then offer some examples of professional learning activities that leverage the research. In this case, the topic is cognitive demand.

**What is the Cognitive Demand of a Mathematical Task?** We can think of cognitive demand of a mathematical task as the level of intellectual activity that is required of students to complete a particular task. Studies have shown that students who regularly engage with cognitively demanding mathematical tasks perform better on tests of problem solving and mathematical reasoning (NCSE, 2003; Stein & Lane, 1996; Tarr et al., 2008). Tasks that involve rote memorization and reproduction of procedures would be considered lower in cognitive demand compared to those that ask students to understand the concepts behind the procedures. Expectedly, solving genuine problems, conjecturing, and proving would be considered higher-level engagement with mathematics.

**Different Levels of Cognitive Demand.** Much of the research about cognitive demand of tasks stems from a study by Henningsen and Stein (1997). The authors developed the Mathematical Task Framework wherein they break down the cognitive demand of a mathematical task into four different levels (from lowest to highest): memorization, procedures without connection, procedures with connection, and doing mathematics. *Memorization* tasks typically consist of a recall of a mathematical fact, formula, definition. *Procedures Without Connections* tasks involve following a procedure without having to understand its underlying concepts. Such tasks do not typically demand an explanation of the procedures. *Procedures With Connections* tasks focus on using a procedure in order to build understanding of its underlying concepts. Such connections can be built as a result of connecting different representations of the problem or the solution. Lastly, *Doing Mathematics* tasks usually involve mathematically justifying approaches to open-ended problems for the purpose of building conceptual understanding. It also includes cases when students invent new approaches, or better yet, new mathematics!

**Changes in Levels of Cognitive Demand During Instruction.** The research about this framework has highlighted that the cognitive demand of a task can start at one level as written but change during instruction. The more common pattern that different studies have documented is that high level tasks often turn into a routine task with minimal demand during instruction (Henningsen & Stein, 1997; Romanagno, 1994; Stein, Grover & Henningsen, 1996). That is, as an

effort to support their students, instructors often end up lowering the demand by giving too many hints or removing the open-endedness of a problem. Selecting "good" problems to work with students is only part of the work, maintaining or even raising the cognitive demand is another. The question then becomes, how does one maintain, or raise, the level of cognitive demand?

Research that focuses on this phenomenon of changes in cognitive demand has identified some factors that led to the maintenance of cognitive demand, including: giving students appropriate amount of time, sustaining pressure for explanation and meaning, and the task building on students' prior knowledge (Henningsen & Stein, 1997). The instructor can also model high-level engagement by modeling self-monitoring (noting progress and remaining issues), asking questions that draw out conceptual connections, and problematizing. Problematizing can involve treating previously accepted facts as examinable claims, common explanations as needing evidence, and standard procedures as needing justifications (Engle & Conant, 2002). Some studies have identified problematizing as a teacher move to help increase the cognitive demand of mathematical tasks during instruction (Engle & Adiredja, 2007).

**Activities for Professional Learning About Teaching Related to Cognitive Demand.** Note that these are activities for which the learners are *people who teach college mathematics*. The tasks might be used in a workshop or seminar for novice college mathematics instructors (e.g., graduate students learning to teach undergraduate mathematics). Each activity is based on resources that are publicly available.

**Activity 1:** Characterizing the Cognitive Demands of Mathematical Tasks: A Task-sorting Activity

- **Full reference:** Stein, M. K., Smith, M. S., Arbaugh, F., Brown, C. A. & Mossgrove, J. (2004). Characterizing the cognitive demands of mathematical tasks: A task-sorting activity. *Professional development guidebook* (A supplement to the National Council of Teachers of Mathematics 2004 Yearbook). Reston, VA: National Council of Teachers of Mathematics. Retrieved from North Dakota Department of Public Education website: `https://www.nd.gov/dpi/uploads/1382/CharacterizeCognitiveDemandTaskSortingActivityarticle.pdf`
- **Goals:** Developing the skill and awareness to identify differences in cognitive demand of tasks, and engaging novice instructors to analyze and discuss tasks.
- **Connection to cognitive demand:** This activity will help familiarize instructors with the different levels of cognitive demand discussed in the research summary.
- **Synopsis:** Participants will begin by solving the tasks themselves, then leaders will facilitate a whole group discussion about different strategies and solution paths. Once finished, the instructors would sort the activities according to their levels of cognitive demand using the complete guide for cognitive demand, which can be found in the MAA IP Guide on page 31 – `https://www.maa.org/sites/default/files/InstructPracGuide_web.pdf`.

**Activity 2:** Video Cases for College Mathematics Instruction–Office Hours

- **Full reference:** Hauk, S., Speer, N. M., Kung, D., Tsay, J.-J., & Hsu, E. (Eds.) *Video cases for college mathematics instructor professional development*, Office Hours Case. Retrieved from `http://collegemathvideocases.org/cases/case.php?VCID=5`
- **Goals:** The goal of the designed activity is to develop awareness of questions to ask for instructional diagnosis in office hours, and to practice noticing and probing student thinking.

**Connection to cognitive demand:** This activity will engage instructors in thinking about how they would react to students' questions and current state of mathematical understanding. In this way, the activity will provide an opportunity for instructors to be thinking about some of the factors that can help maintain or increase cognitive demand (e.g., problematizing). Instructors can also reflect on the temptation to resolve issues for students as they are struggling, which would decrease the cognitive demand of the mathematical task.

**Synopsis:** Participants will solve the problems that students are asking questions about in the video. They will then watch the video of office hour interactions and discuss with facilitators using the discussion questions listed. There are some reflection questions after watching the video, which can be done individually or in a group setting.

## References


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## About the Author


Aditya Adiredja `<adiredja@math.arizona.edu>` is an assistant professor of mathematics at Arizona State University. His research lies in the intersections between advanced mathematics, equity and cognition. He investigates ways that students make sense of challenging mathematical topics in undergraduate curriculum with a particular interest in the role of intuitive knowledge in learning formal mathematics. He explores ways that our views of knowledge and learning determine what kind of knowledge and what kind of students get privileged in the classroom.